Lecture 6

Thm $f : M_A \to M_A$ p.h. s.t.
$\nexists$ f-periodic T tang to E^{cs} or E^{cu}
$\Rightarrow$ f is dynamically coherent
and (an iterate) is leaf conj to
the susp. of an Anosov.

$A \in SL(2,\mathbb{Z})$
hyp.

$$M_A = \mathbb{T}^2 \times [0,1] \Big/ (x,1) \sim (Ax,0)$$

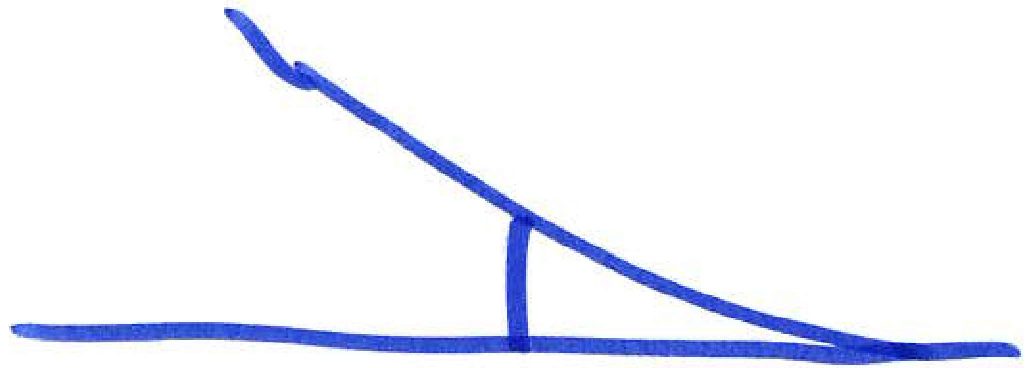
Anosov flows:

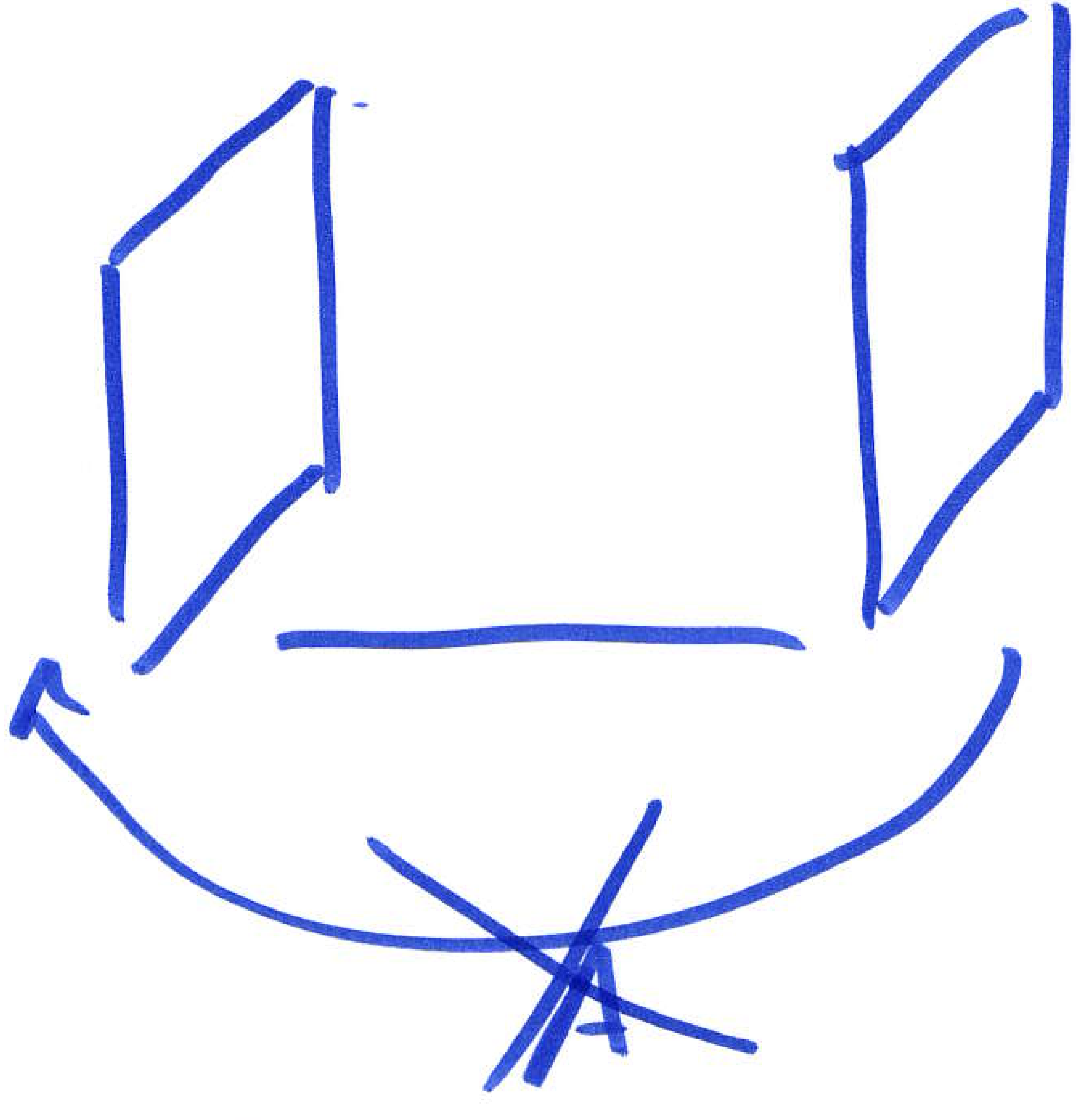
→ center leaves must be fixed

→ no periodic points in the universal cover.

<u>Tools</u>: 1) Classify foliations without tori (leaves separate)

2) Mapping class group in M_A is finite





$B \times id$

$$\tilde{M}_A = \mathbb{R}^2 \times \mathbb{R}$$

$$\gamma_1(x,t) = (x+(1,0),t)$$

$$\gamma_2(x,t) = (x+(0,1),t)$$

$$\gamma_3(x,t) = (Ax,t-1)$$

$$\Phi_s(x,t) = (x, t+s)$$

$$\mathcal{A}^{cs} = E^s_A \times \mathbb{R}$$

$$\mathcal{A}^{cu} = E^u_A \times \mathbb{R}$$

Prop: Every $\mathcal{F}$ foliation w.o. tori of M_A is Almost parallel to either $\mathcal{A}^{cs}$ or $\mathcal{A}^{cu}$.

Th (BI) $\exists\ \mathcal{F}^{cs}$ and $\mathcal{F}^{cu}$ f-inv branching foliations which are almost parallel to true foliations

$\mathcal{F}^{cs}$ is AP either $\mathcal{A}^{cs}$ or $\mathcal{A}^{cu}$

$\mathcal{F}^{cu}$ is AP " $\mathcal{A}^{cs}$ or $\mathcal{A}^{cu}$

<u>Assumption</u>: $\mathcal{F}^{cu}$ is $\underset{\substack{\uparrow \\ R}}{AP}$ to $\mathcal{A}^{cu}$

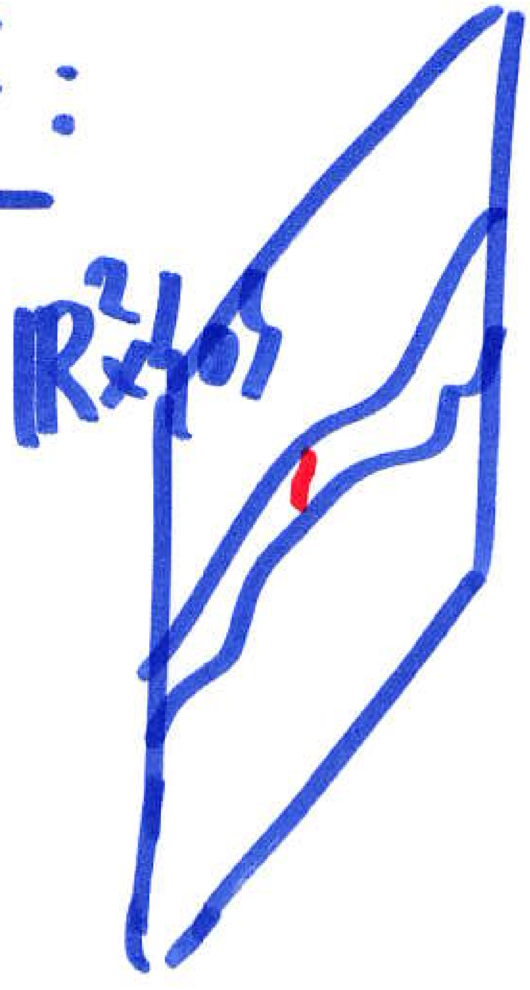
[<u>1st GOAL</u>: Show $\mathcal{F}^{cs}$ is AP to $\mathcal{A}^{cs}$]

Choose $\tilde{f}$ a lift $d(\tilde{f}, id) < K_0$

For a leaf $\mathcal{L}$ of $\mathcal{A}^{cu}$ consider $\Gamma \subseteq \tilde{\mathcal{F}}^{cu}$ the set of leaves at dist. $< R$ from $\mathcal{L}$.

Lemma Γ is a unique leaf.
(no Denjoy)

Pf:



A^{cu} is fixed by a deck tr.

$(x,t) \mapsto (x+n, t)$

Slice $\times [P(n), p(n)]$

$\blacksquare$

Corollary: Every surface of $\tilde{\mathcal{F}}^{cu}$ is fixed by $\tilde{f}$.

$$\tilde{\mathcal{F}}^{cu} \longleftrightarrow \tilde{\mathcal{F}}^{cs}$$

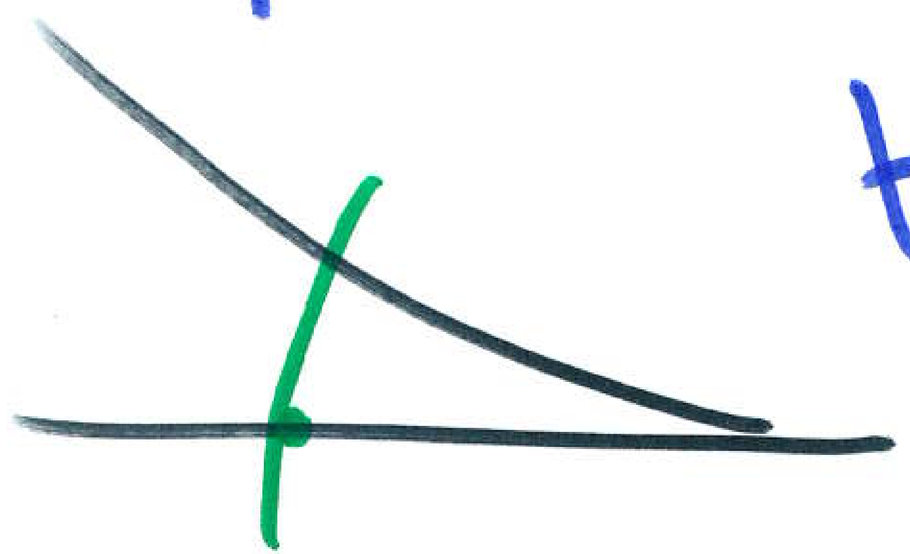
$$p : \mathbb{R}^2 \times \mathbb{R} \longrightarrow \mathbb{R}$$

$$(x, t) \longmapsto t$$

Crucial lemma $\exists K_1$ such that $\forall x \in \mathbb{R}^3$ we have that

$$p(\tilde{f}^n(x)) - p(x) > -K_1$$

$$\forall n \geq 0$$



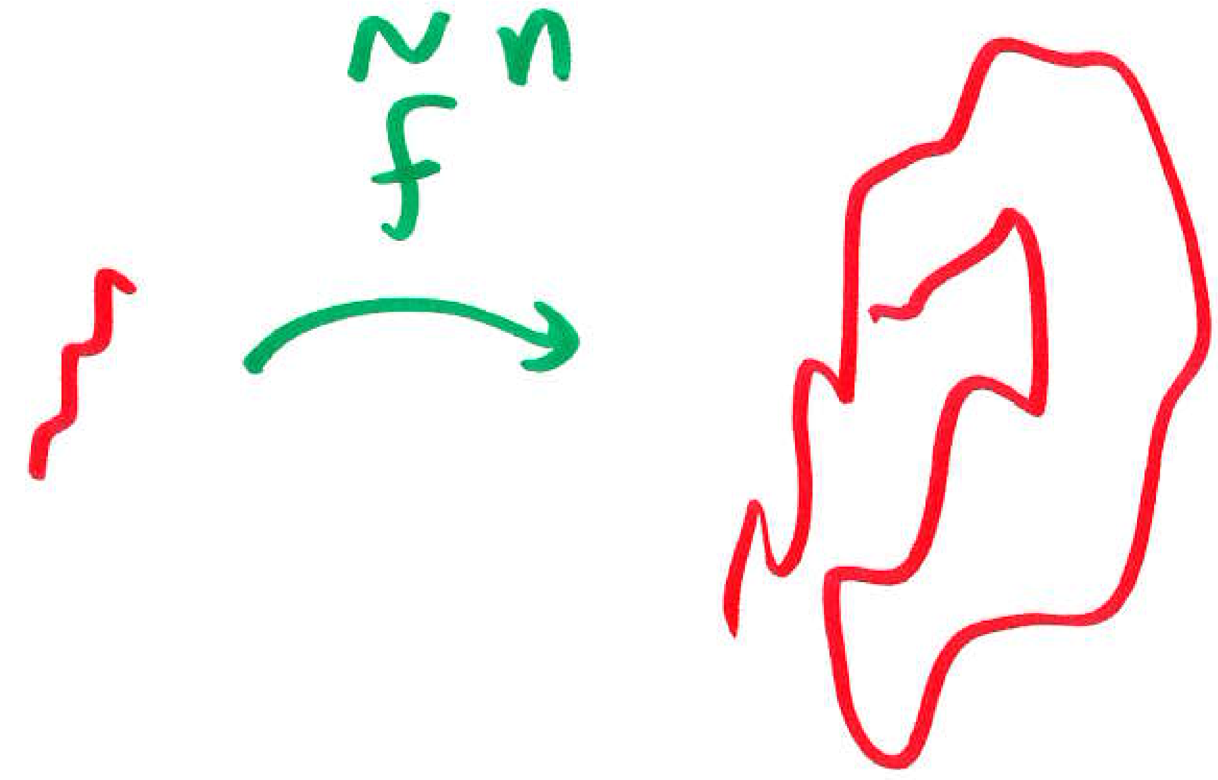
<u>Prop</u>: $\tilde{\mathcal{F}}^{cs}$ is AP to $\mathcal{A}^{cs}$.

<u>Pf</u>: Assume $\mathcal{F}^{cs}$ AP to $\mathcal{A}^{cu}$

$$\forall x \in \mathbb{R}^3 \quad p(\tilde{f}^{-n}(x)) - p(x) > -K_1 \quad \forall n \geq 0$$

$$\Rightarrow \forall x \in \mathbb{R}^3 \ \forall n \in \mathbb{Z}$$

$$-K_1 < p(\tilde{f}^{n}(x)) - p(x) < K_1$$

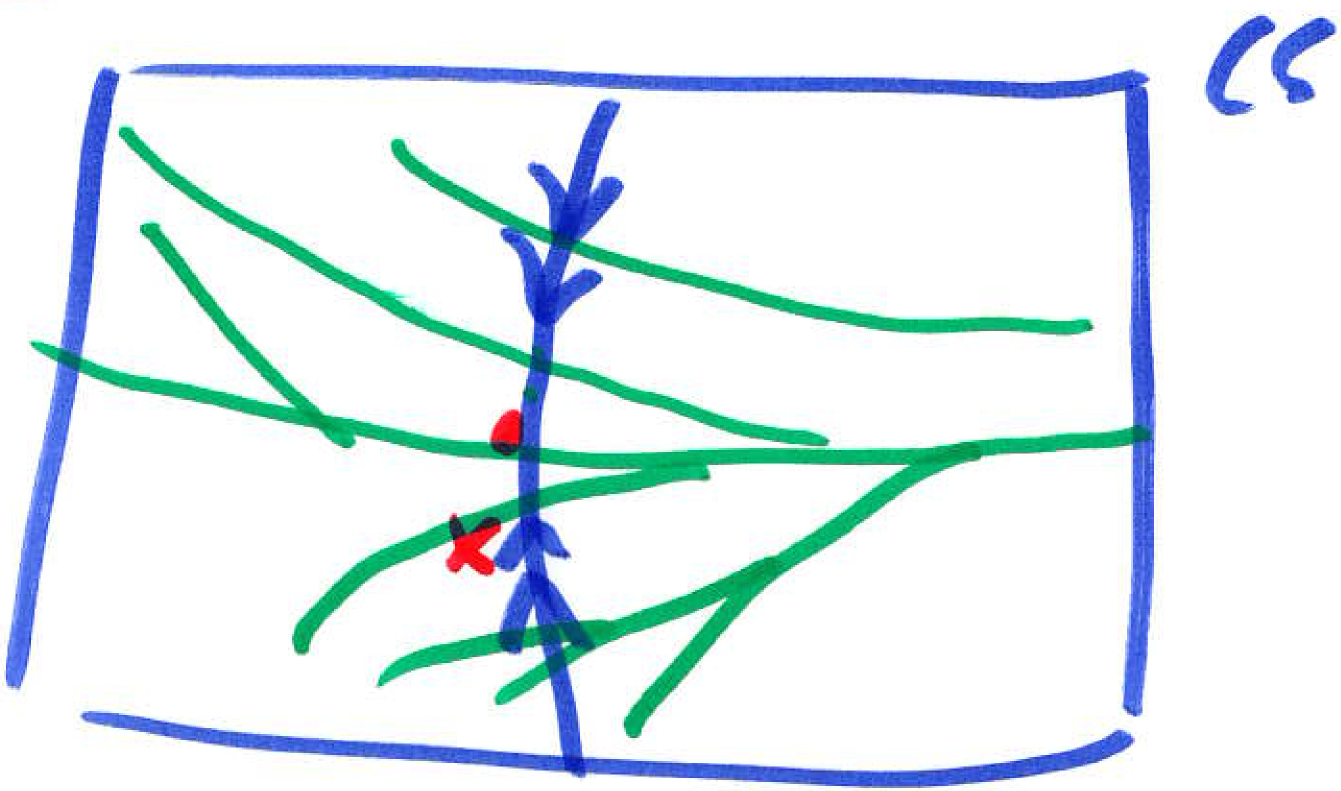


Novikov Thm.

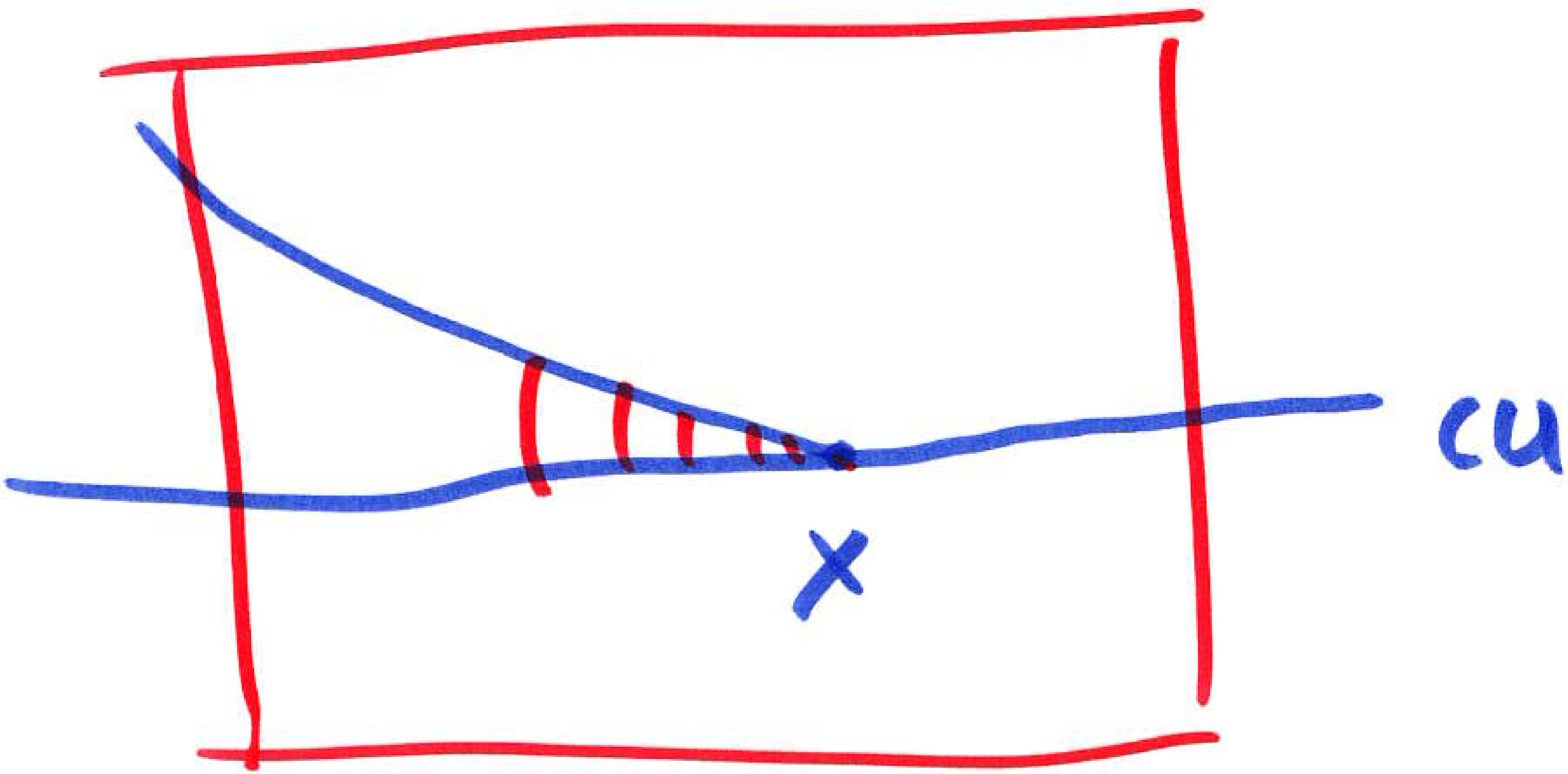
Consequence (modulo iterate)

Every intersection between cs-surfaces and cu-surfaces is fixed by $\tilde{f}$.

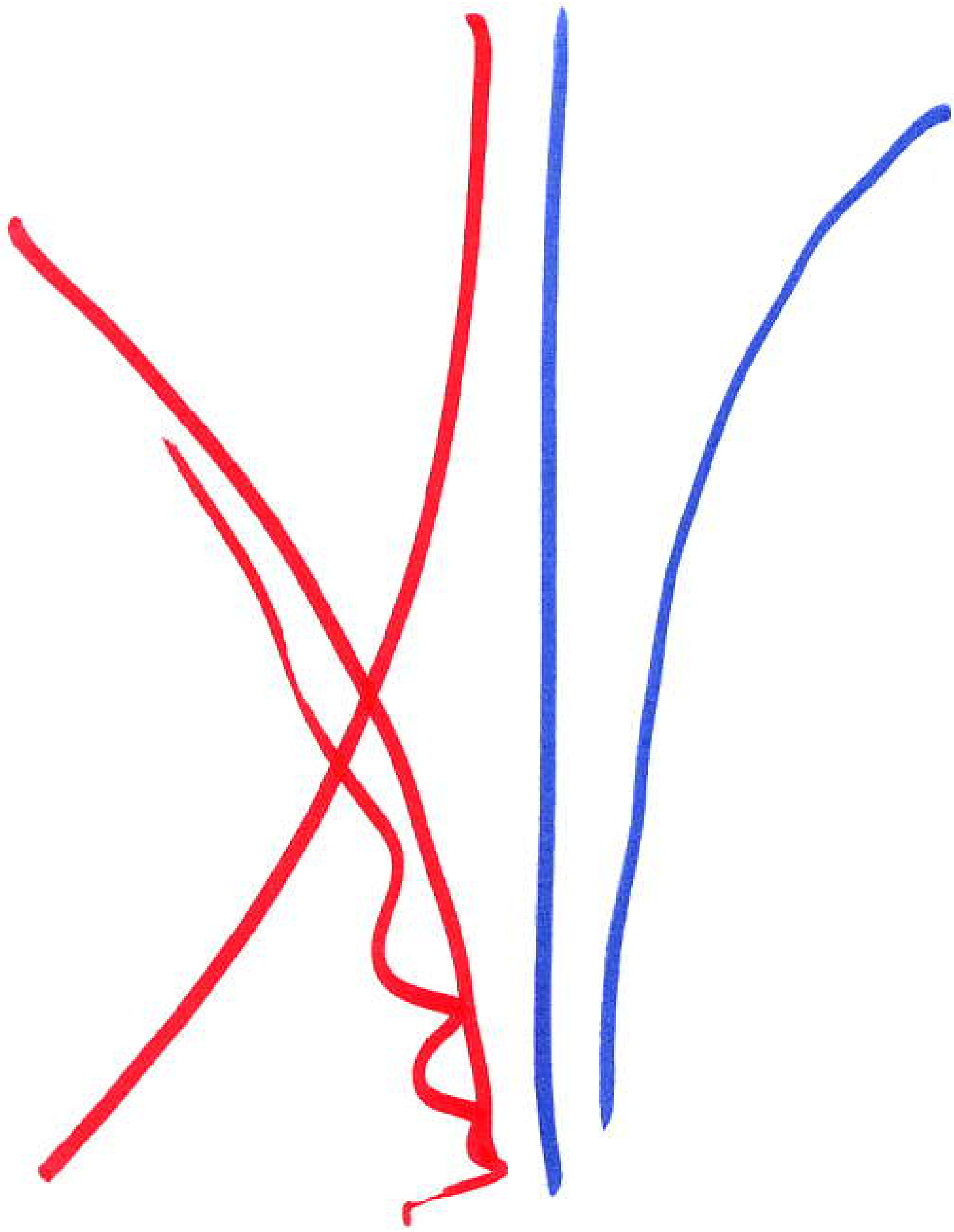
BW argument : No periodic points in the universal cover



Dynamical coherence:



$\Rightarrow$ OK + uniform advance in flow direction



<u>Conj</u>: Every p.h. $\left\{\begin{array}{l}\text{transitive} \\ \text{dyn.coh.} + \dim M = 3\end{array}\right\}$

leaf. conj. is

→ Anosov

→ Skew product

→ Anosov flow.