$f: M \circlearrowleft$ p.h. M 3-mfld w solv fund gp

$M = T^3$ f has hyp linear part	$M = T^3$ non-hyp linear part
M is a non-trivial S^1-bundle over T^2	$M = M_A$ is a suspension manifold.

LECTURE 5

<u>Thm</u> If $f : \mathbb{T}^3 \circlearrowleft$ is p.h. and its linear part $A : \mathbb{T}^3 \circlearrowleft$ is Anosov, then

f is leaf conj to A.

M

$(f, W^c_f) \qquad (g, W^c_g)$

A <u>leaf</u> <u>conj</u> is a homeo

$h : M \to M$ s.t.

$$L \in W^c_f \Longrightarrow h(L) \in W^c_g$$

and

$$hf(L) = gh(L).$$

Thm If $f : \mathbb{T}^3 \circlearrowleft$ p.h. with hyperbolic linear part $A : \mathbb{T}^3 \circlearrowleft$ then f is leaf conj to A.

$\tilde{M} = \mathbb{R}^3$ $\quad f : \mathbb{R}^3 \to \mathbb{R}^3$

linear Anosov $A : \mathbb{R}^3 \to \mathbb{R}^3$

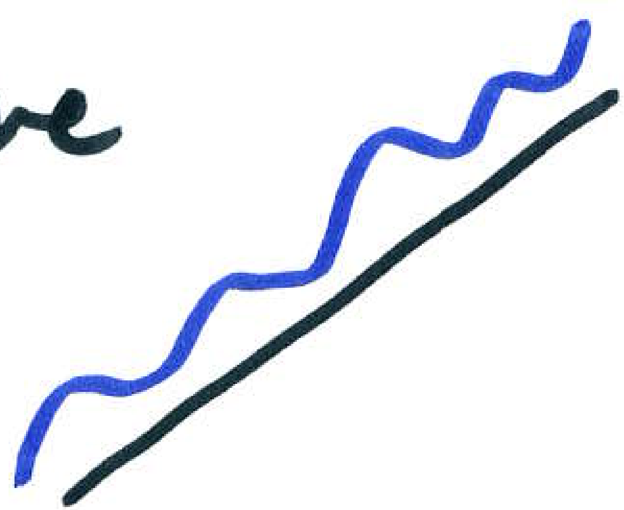
and $\quad \mathrm{dist}_{C^0}(f, A) < \infty$

$$T\mathbb{R}^3 = \underbrace{E^u \oplus \rlap{$\underbrace{}_{W^{cs}}$}E^c}_{W^{cu}} \oplus E^s$$

linear splitting

$$T\mathbb{R}^3 = E^u_A \oplus E^c_A \oplus E^s_A$$

W^{cs}_f | $\exists R$ s.t. every leaf of W^{cs}_f is R-close to a linear cs-plane



Same holds for W^{cu}_f

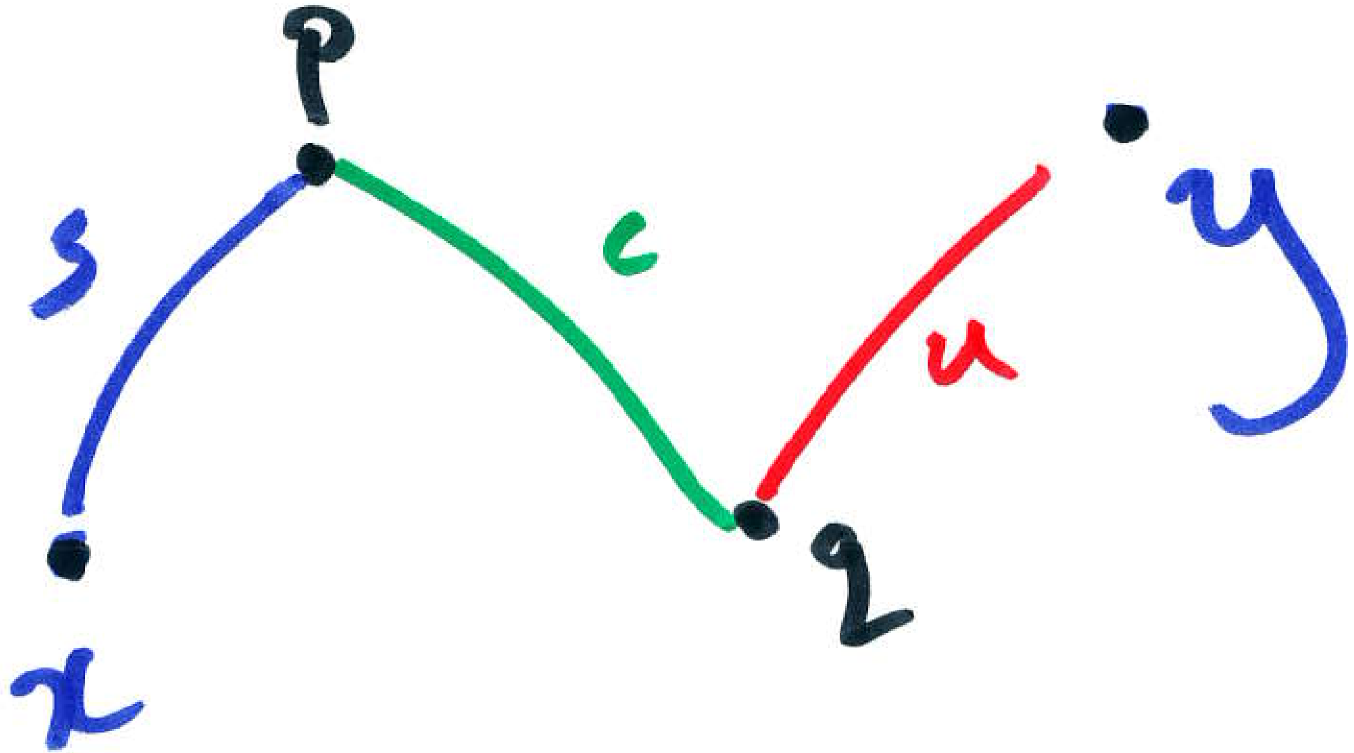
so by intersecting

leaves of W^c_f are

R-close to

linear c-lines.

GPS: $\forall x, y \in \tilde{M} \; \exists!\, p, q \in \tilde{M}$



C_f the space of center leaves on $\tilde{M} = \mathbb{R}^3$

$C_A = \mathbb{R}^3 / E^c_A \cong \mathbb{R}^2$

$C_f \cong \mathbb{R}^2$

Build homeo $H : C_f \to C_A$ s.t.

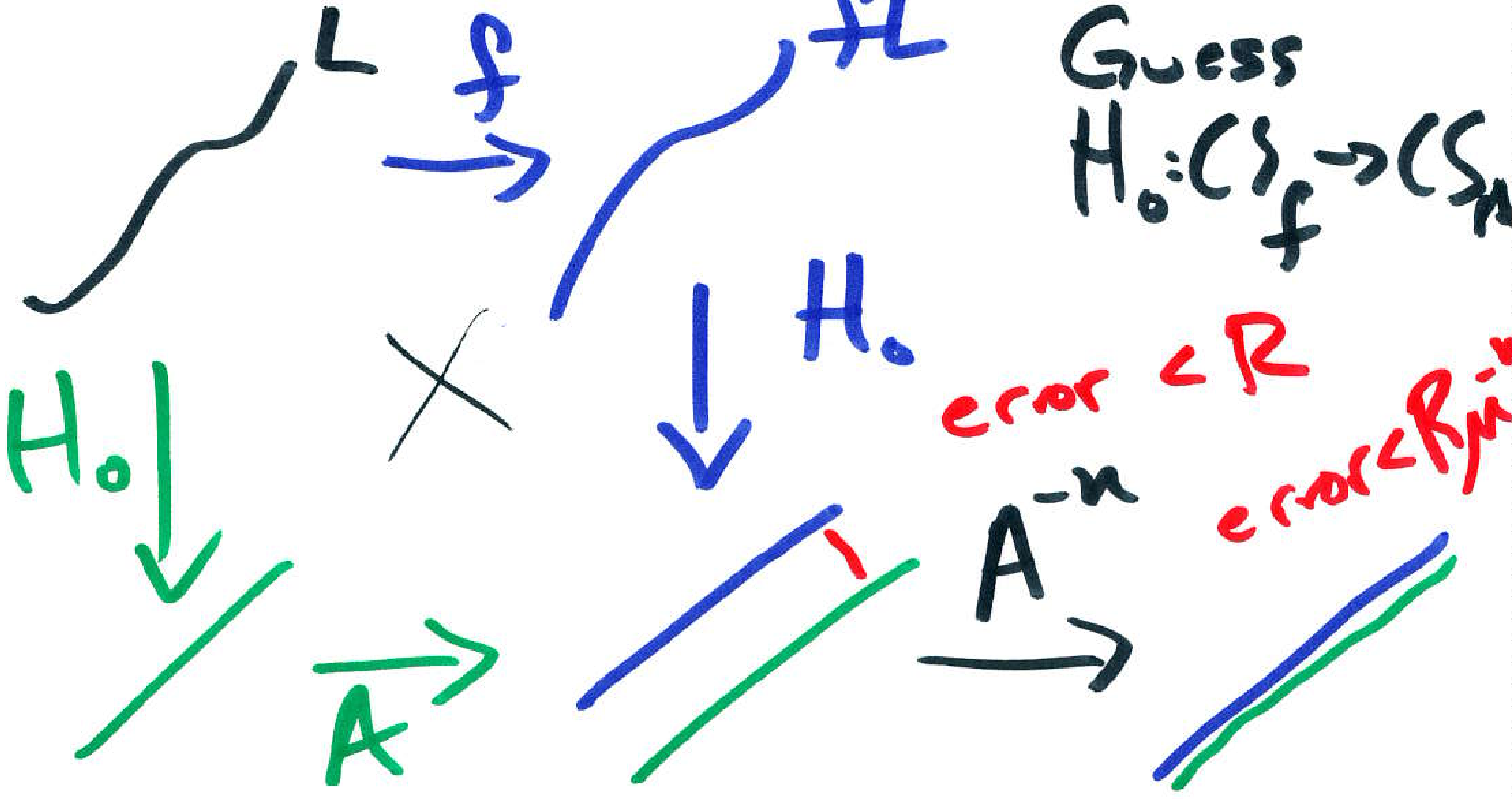
$Hf = AH$.

$CS_f \longrightarrow CS_A$

$H^{(3)}: CS_f \longrightarrow CS_A$

Guess

$H_0 : CS_f \to CS_A$



$H_0 \,,\quad H_n := A^{-n} H_0 f^n$

$H_n \longrightarrow H^{cs}$ error $\to 0$

so $\quad H^{cs} f = \cancel{B} A H^{cs}$.

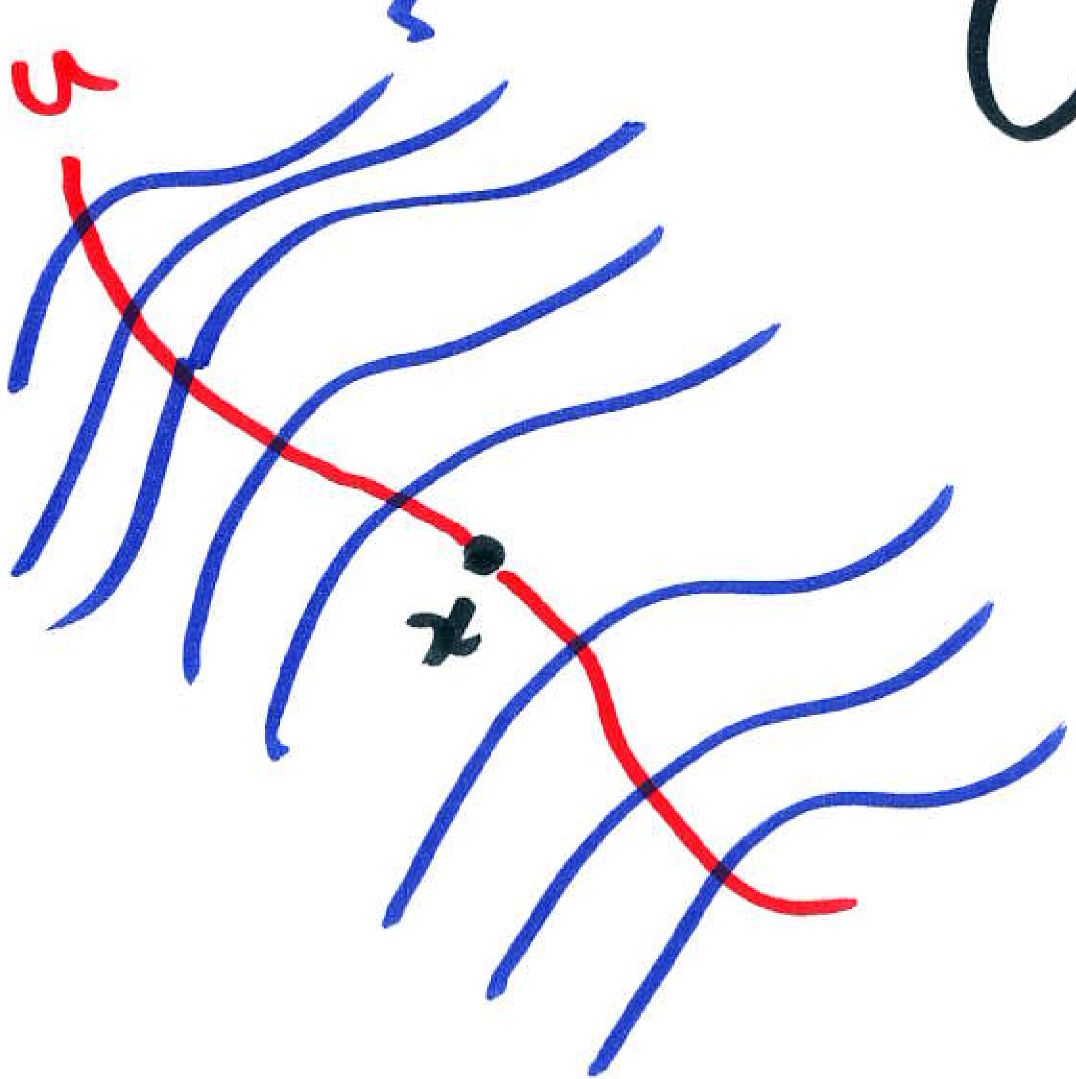
$CS_f \xrightarrow{H^{cs}} CS_A$

$CU_f \longrightarrow CU_A$

$H : C_f \to C_A$

$Hf = AH$.

us-pseudoleaf



$$W^{us}_f(x) = \bigcup_{y \in W^u_f(x)} W^s_f(y)$$

C^0 plane.

GPS $\Rightarrow$ intersects each center leaf exactly once

Want a section Σ s.t.

- intersecting each c-leaf once
- unif cts
- finite dist from P

$\tilde{M}$ Heisenberg space.

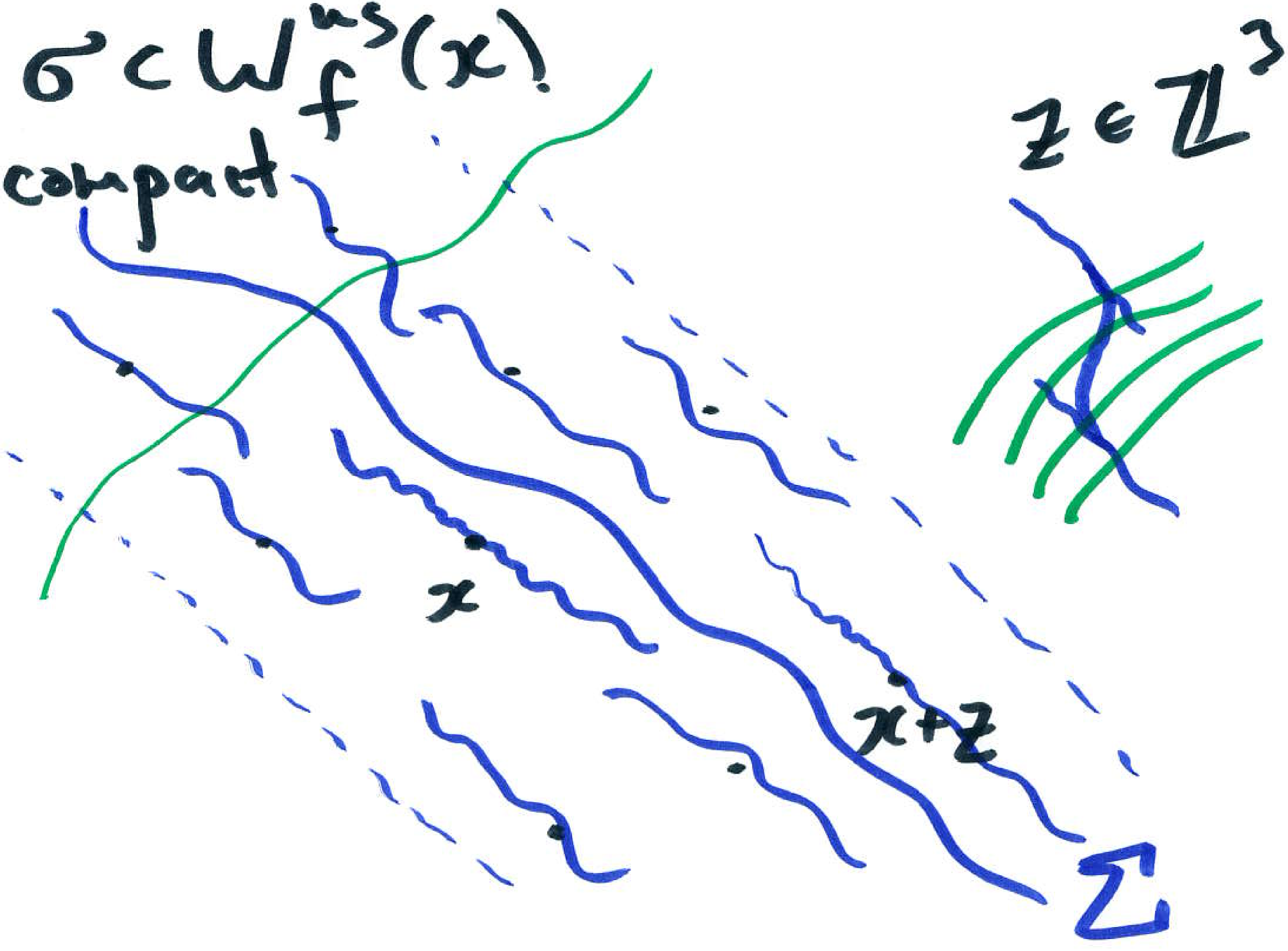
$f : \tilde{M} \circlearrowleft$ ~~$\mathrm{dist}_0($~~,

$\Phi : \tilde{M} \circlearrowleft$ $\mathrm{dist}_0(f, \Phi)$

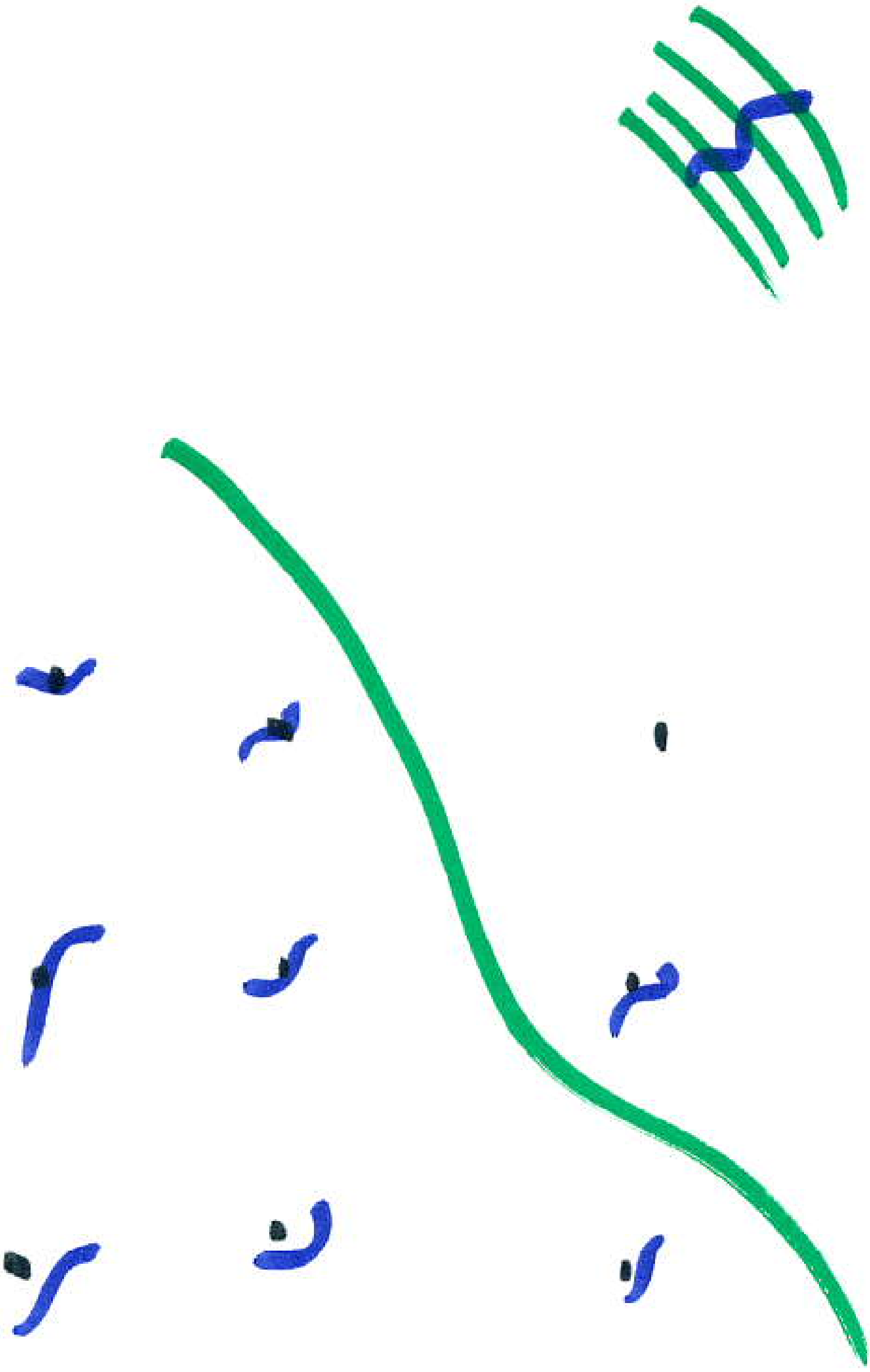
lie gp homo
aut.

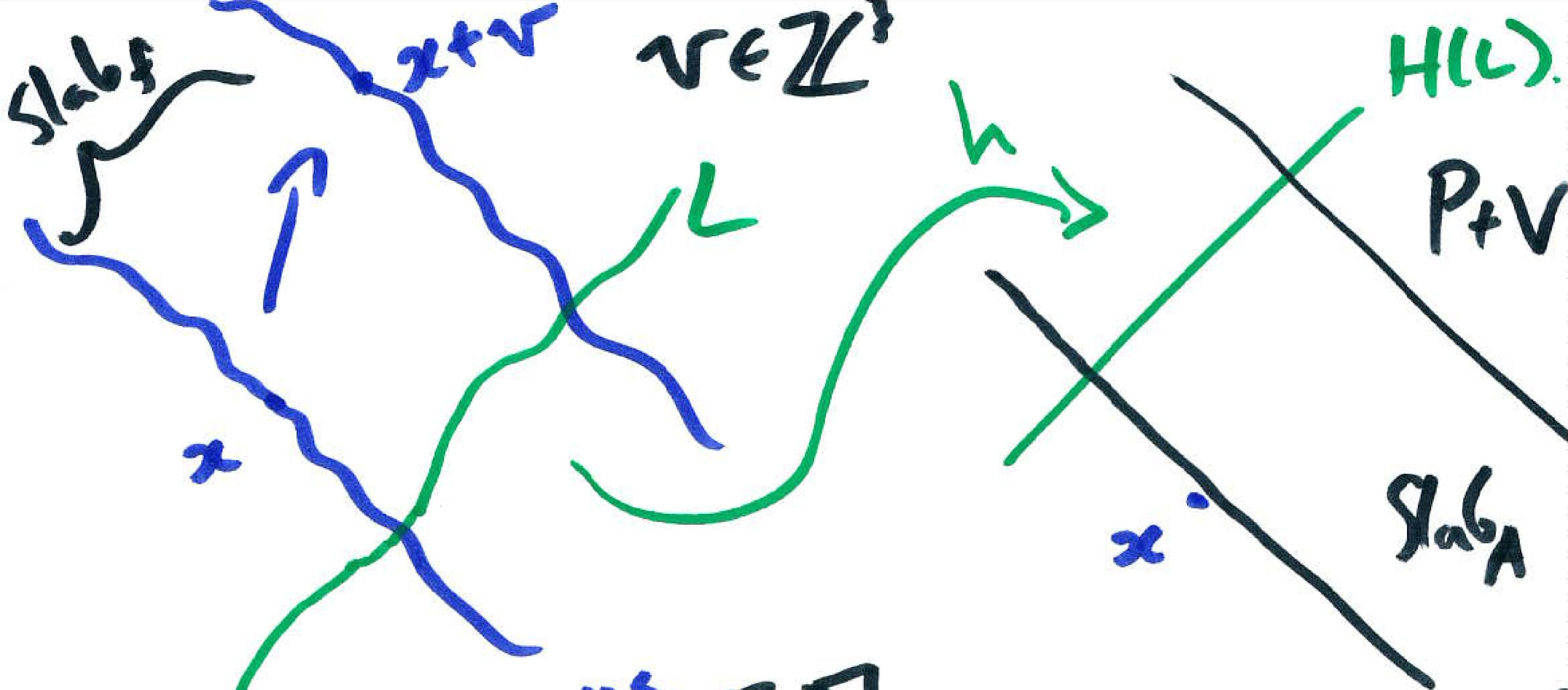
There are coordinates (x, y, z) for $\tilde{M}$ s.t.

$$\Phi(x, y, z) = (\lambda^{-1} x, \lambda y, z).$$



σ ⊂ W^us_f(x)
compact
z ∈ ℤ³
x
x+z
Σ





$\cancel{W^{us}_f(x)}$ Σ

$P = W^{us}_A(x)$

Don't know if $W^{us}_f(x)$ is unif cts finite dist from P.

$\Pi^3 \to \Pi^3$

$h : \mathrm{Slab}_f \longrightarrow \mathrm{Slab}_A$

extend to $\mathbb{R}^3$ by

$$h(x+v) = h(x) + v.$$

$h : \mathbb{R}^3 \circlearrowleft$ leaf conj.

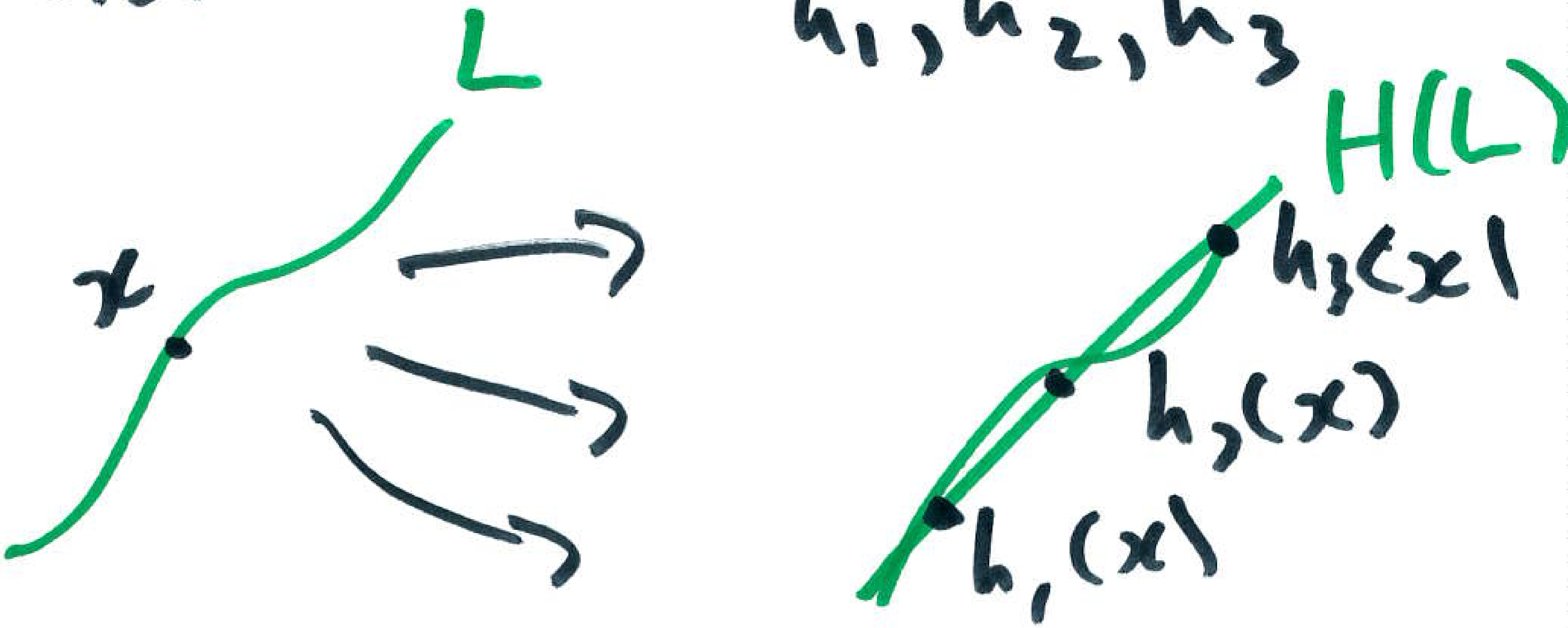
$h_z :$ $\quad h_z(x+z) = h(x) + z.$ $\quad (z \in \mathbb{Z}^3)$

Family of leaf conj

$\uparrow \quad \mathcal{H}_0 = \{ h_z : z \in \mathbb{Z}^3 \}.$

equicts.

h_1, h_2, h_3



Define average $\frac{1}{3}(h_1+h_2+h_3)$.

$\mathcal{H}_1 = \{$all averages of elts in $\mathcal{H}_0\}$.

equiv. cts.

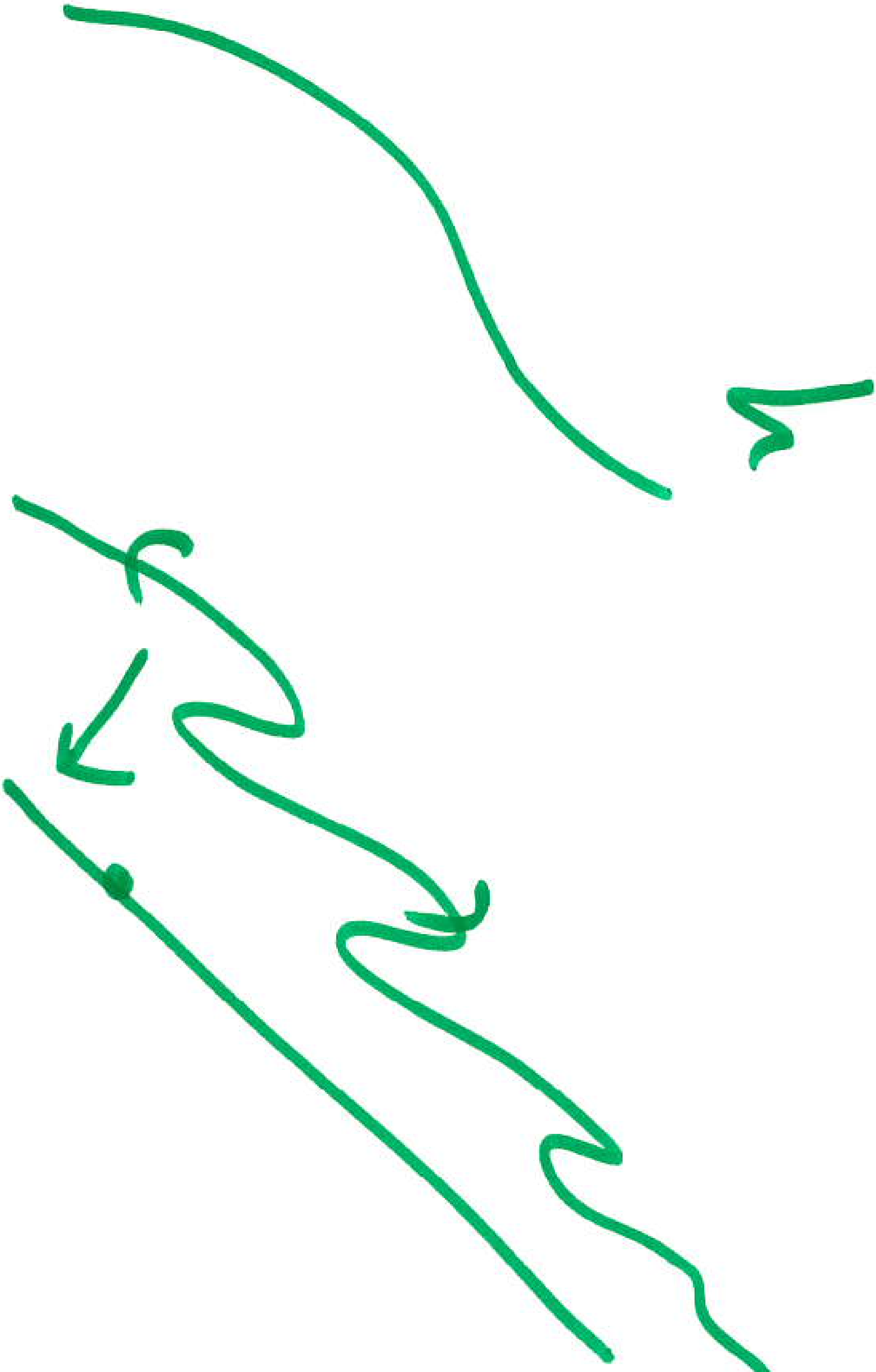
$$\mathrm{Cube}(N) = \{(i,j,k) : |i|, |j|, |k| < N\}$$

$$h_n = \frac{1}{\#\mathrm{Cube}(N)} \sum_{z \in \mathrm{Cube}(N)} h_z.$$

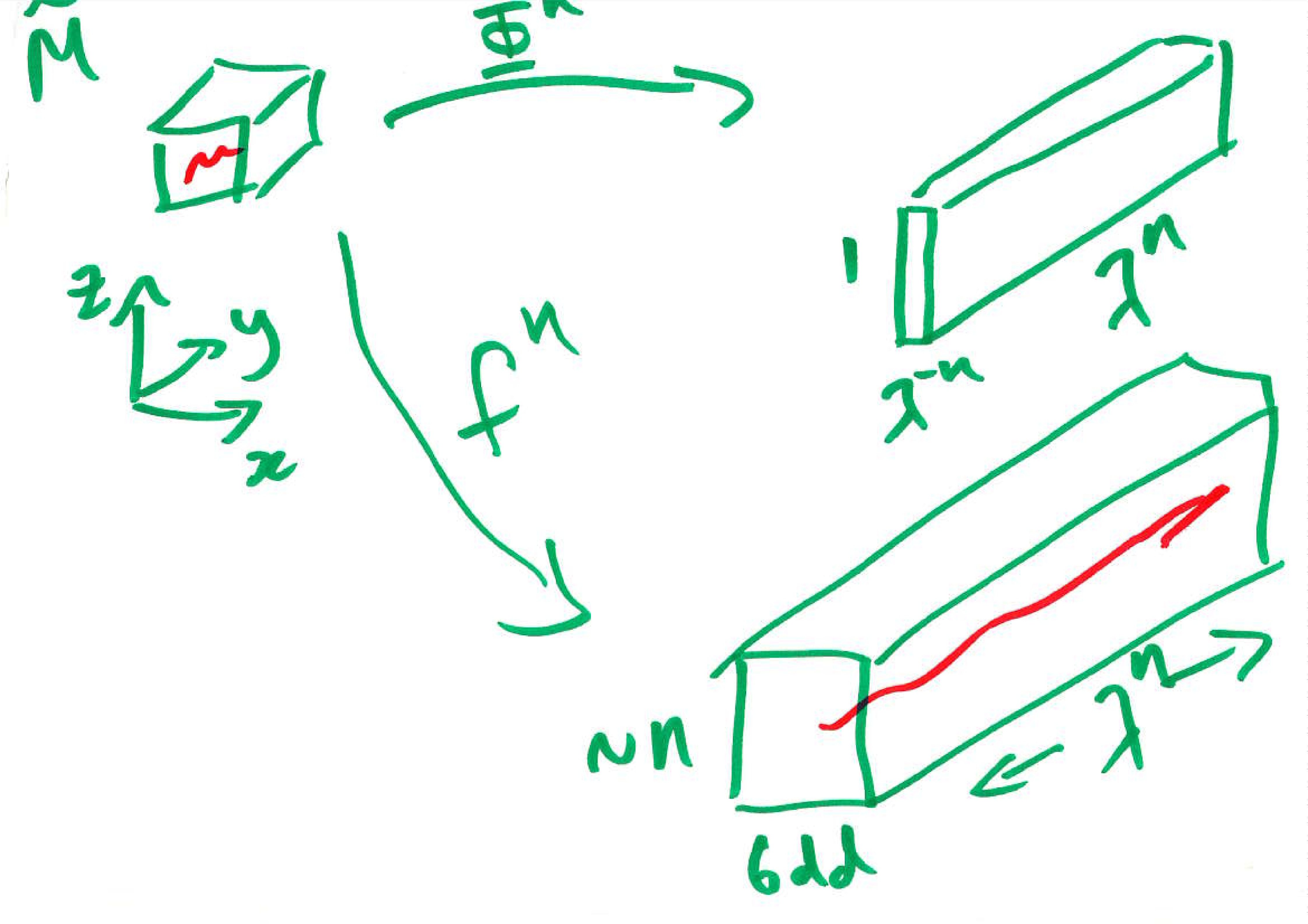
Arzela-Ascoli: $\exists n_k$

$$h_{n_k} \longrightarrow h_\infty$$

h_∞ descends to a leaf conj on $\mathbb{T}^3$.



<u>Thm</u> If $f: M \circlearrowleft$ p.h. and M is a non-triv S^1-bundle over $\mathbb{T}^2$ then f is a skew
topological
product



$\tilde{M}$
Φ^n
f^n
x
y
z
1
λ^n
λ^{-n}
λ^n
$\sim n$
$6dd$