Dyck Paths in Representation Theory of Algebras.

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In 1971 [4] Coxeter introduced the friezes patterns like an array such that the first and the last row are zeroes, and every four adjacent numbers form the following square

$$\begin{array}{ccc} & b & \\ a & & c \\ & d & \end{array}$$

which satisfy the equation $ac - bd = 1$. Conway and Coxeter [3] presented a connection between the frieze patterns and the triangulation of polygons using the first row different of ones and the number of triangles incident to a vertex. Caldero, Chapoton and Schiffler [2] introduced the category of diagonals $\mathcal{D}_n$ of the $(n+3)$ polygon for which indecomposable objects are the diagonals of the polygon and the irreducible morphisms are direct sums of elementary movements between diagonals. They proved that this category is equivalent to the cluster category in the case $\mathbb{A}_n$. In this talk, we describe a categorical equivalence between category $\mathcal{D}_n$ and a new combinatorial category with indecomposable objects defined by Dyck paths. For this category, we also describe the irreducible morphisms, the Auslander-Reiten quiver and we present some connection between Dyck paths, friezes, and triangulations associated to algebras of type $\mathbb{A}_n$.

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