LARGE HARMONIC FUNCTIONS FOR FULLY NONLINEAR FRACTIONAL OPERATORS

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Abstract. We study existence, uniqueness and boundary blow-up profile for fractional harmonic functions on a bounded smooth domain $\Omega \subset \mathbb{R}^N$. We deal with harmonic functions associated to uniformly elliptic, fully nonlinear nonlocal operators, including the linear case

$$(-\Delta)^s u = 0 \quad \text{in } \Omega,$$

where $(-\Delta)^s$ denotes the fractional Laplacian of order $2s \in (0, 2)$. We use the viscosity solution's theory and Perron's method to construct harmonic functions with zero exterior condition in $\bar{\Omega}^c$, and boundary blow-up profile

$$\lim_{x \to x_0, x \in \Omega} \mathrm{dist}(x, \partial\Omega)^{1-s} u(x) = h(x_0), \quad \text{for all} \quad x_0 \in \partial\Omega,$$

for any given boundary data $h \in C(\partial\Omega)$. Our method allows us to provide blow-up rate for the solution and its gradient estimates. Results are new even in the linear case.

Joint work with Gonzalo Dávila and Erwin Topp.

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