DYNAMICS AND COMBINATORICS OF BRANCHED COVERINGS

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The critical orbit of a rational function, acting on the Riemann sphere by iteration, has a determining effect on the global dynamics. A very special class is the one formed by those functions with all critical orbits finite, known as postcritically finite functions. Thurston provided a remarkable characterization of postcritically finite rational functions into the class of floppy rational functions, the postcritically finite branched self-coverings of the two-sphere, called Thurston maps (1983). In the 2010s, he was considering again the question of understanding holomorphic mappings from the topological point of view. At that time, he introduced the balanced planar 4-regular graphs and showed that they combinatorially characterize all cell graph $\Gamma = f^{-1}(\Sigma) \subset \mathbb{S}^2$, where $f : \mathbb{S}^2 \to \mathbb{S}^2$ is an generic orientation-preserving degree d branched covering, and $\Sigma \subset \mathbb{S}^2$ is an oriented Jordan curve passing through the critical values of f (the word generic means that the cardinality of the set of critical values of f is $2d-2$, the largest possible).

In this course, we will go over this two topics:

- Firstly, we will study the basic theory of complex dynamics and Thurston maps including some applications and new perspectives in the Theory. We intend to provide a clear exposition of the mean ideas involved in the theory via detailed proof sketch of Thurston's characterization of rational functions theorem;
- Secondly, we will see that the most natural generalization of the balance condition for higher genera does not suffice for the realizability of a cell graph as a pullback graph Γ. Then, with one more natural imposition, we provide a general version of Thurston's theorem, which encompasses every branched covering of the two-sphere.Then, we will introduce and go over some operations defined on the (generalized) balanced graphs, mention some further results, and stress linkages with some distinct mathematical topics.

(first class) Introduction to Thurston maps theory:
(second class) The Thurston classification theorem and a sketch of a proof of it:
(third class) Applications of Thurston's theorem, generalizations and new perspectives:
(fourth class) The combinatorial-topology of branched coverings

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