Statistical Methods and Calibration in Finance and Actuarial Science

Corrections to the Lecture Notes - MA6622[1]

2 Predictability of Asset returns

- Page 11, line 4, says:

$$S_n = \exp(h_1 + \cdots + h_n), \qquad n \geq 1,$$

Should say

$$S_n = S_0 \exp(h_1 + \cdots + h_n), \qquad n \geq 1,$$

3 Predictabiliy of Asset Returns (second part)

- Page 8, line 3, says:

$$B_n = (1 + r)^n$$

Shoud say

$$B_n = B_0(1 + r)^n$$

- Page 9, line -3, says:

$$\mathbf{E_Q}\left(\frac{S_{n+1}}{B_{n+1}} \mid \mathcal{F}_N\right) = \frac{S_n}{B_n},$$

Should say:

$$\mathbf{E_Q}\left(\frac{S_{n+1}}{B_{n+1}} \mid \mathcal{F}_n\right) = \frac{S_n}{B_n},$$

- Page 11, line -2, says:

$$S_n = \exp(h_1 + \cdots + h_n), \qquad n \geq 1,$$

Should say:

$$S_n = S_0 \exp(h_1 + \cdots + h_n), \qquad n \geq 1,$$

- Page 14, line 5, says:

$$\sigma_n^2 = \alpha_0 + \alpha_1 h_{n-1}^2 + \cdots + \alpha_q h_{n-q}^2 + \beta_1 \sigma_1^2 + \cdots + \beta_p \sigma_{n-p}^2,$$

Should say:

$$\sigma_n^2 = \alpha_0 + \alpha_1 h_{n-1}^2 + \cdots + \alpha_q h_{n-q}^2 + \beta_1 \sigma_{n-1}^2 + \cdots + \beta_p \sigma_{n-p}^2,$$

[1] I thank all the student that made corrections and comments on the Notes. Further comments can be send to ernesto.mordecki@gmail.com

4 Markowitz Diversification Portfolio Investment

- Page 10, line -3, says: gives the minimum variance $\sigma^2/4$

  Should say: gives the minimum variance $\sigma^2/2$

- Page 16, line 3, says:

  – If $\beta = 1/2$, the asset A gives half of the expected return of the market, but 1/4 of the systematic risk, we have a defensive asset.

  Should say:

  – If $\beta = 1/2$, the asset A gives half of the risk premium $\mathbf{E}\,\rho_A - r$, with 1/4 of the systematic risk, we have a defensive asset.

5 Portfolio diversification (II) and CAPM

- Page 11, line -4, says: If $\beta = 0$ then $\mathbf{E}\,\rho_A = \rho$.

  Should say: If $\beta = 1$ then $\mathbf{E}\,\rho_A = \rho$.

7 Statistical Fitting with linear models

- Page 14, line 6, says:

$$\bar{W}_{hh} = 1 + 2\big(\bar{\rho}(1)^2 + \cdots + \bar{\rho}(q)\big)$$

  Should say:

$$\bar{W}_{hh} = 1 + 2\big(\bar{\rho}(1)^2 + \cdots + \bar{\rho}(q)^2\big)$$

8 Multivariate linear Time Series

- Page -2, line 5, says: a cross coviance:

  Should say: a cross covariance:

9 Multivariate linear Time Series (II)

- Page 10, lines 1 and 3, says: In order to estimate the matrix ϕ we must solve a matrix equation (i.e. $d \times d$ linear equations) of the form

$$\bar{\Gamma}(1) = \Phi_1 \bar{\Gamma}(0),$$

Should say: In order to estimate the matrix Φ we must solve a matrix equation (i.e. $d \times d$ linear equations) of the form

$$\bar{\Gamma}(1) = \Phi\bar{\Gamma}(0),$$

11 Value at Risk through Monte Carlo

- Page 7, lines 2 and 4, says:

  An alternative approach to the historical **var**, is the simulation or Monte Carlo approach. The main problem that we face when trying to compute **var** is that we do not know . . .

  Should say:

  An alternative approach to the historical VaR, is the simulation or Monte Carlo approach. The main problem that we face when trying to compute VaR is that we do not know . . .

- Page 9, line 8, says:

$$f_\varepsilon(x) = \frac{1}{4}\varphi\Big(\frac{x}{\sqrt{2}}\Big) + \frac{3}{4}\varphi\Big(\frac{x}{\sqrt{2/3}}\Big)$$

  Should say:

$$f_\varepsilon(x) = \frac{1}{4\sqrt{2}}\varphi\Big(\frac{x}{\sqrt{2}}\Big) + \frac{3\sqrt{3}}{4\sqrt{2}}\varphi\Big(\frac{x}{\sqrt{2/3}}\Big)$$

- Page 16, line 1, says: Comments on Derivatives and **var**

  Should say: Comments on Derivatives and VaR

12 Conditional heteroscedastic models (ARCH)

- Page 6, line -1, says:

$$\mathbf{E}(X(t) \mid \mathcal{F}(t)) = \mathbf{E}(\mu + \sigma\varepsilon(t) \mid \mathcal{F}(t-1)) = \mu,$$

  Should say:

$$\mathbf{E}(X(t) \mid \mathcal{F}(t-1)) = \mathbf{E}(\mu + \sigma\varepsilon(t) \mid \mathcal{F}(t-1)) = \mu,$$

- Page 7, lines 3,4,5, says:

$$\begin{aligned}&\mathbf{var}(X(t)\,|\mathcal{F}(t-1))\\ &\qquad\qquad = \mathbf{E}\left[X(t)^2 \mid \mathcal{F}(t-1)\right] - \left[\,\mathbf{E}[X(t) \mid \mathcal{F}(t)]\right]^2\\ &\qquad\qquad = \mathbf{E}\left[(\mu+\varepsilon(t))^2 \mid \mathcal{F}(t-1)\right] - \mu^2 = \sigma^2.\end{aligned}$$

Should say (two missprints):

$$\begin{aligned}&\mathbf{var}(X(t)\,|\mathcal{F}(t-1))\\ &\qquad\qquad = \mathbf{E}\left[X(t)^2 \mid \mathcal{F}(t-1)\right] - \left[\,\mathbf{E}[X(t) \mid \mathcal{F}(t-1)]\right]^2\\ &\qquad\qquad = \mathbf{E}\left[(\mu+\sigma\varepsilon(t))^2 \mid \mathcal{F}(t-1)\right] - \mu^2 = \sigma^2.\end{aligned}$$

- Page 11, line 7, says:

$$\sigma(t)^2 = \sqrt{\mathbf{var}(X(t) \mid \mathcal{F}(t-1))} = \sqrt{\omega + \alpha X(t-1)^2}$$

Should say:

$$\sigma(t) = \sqrt{\mathbf{var}(X(t) \mid \mathcal{F}(t-1))} = \sqrt{\omega + \alpha X(t-1)^2}$$

14 Calibration in Black Scholes Model and Binomial Trees

- Page 6, line 7, says:

$$S(t) = S(0)\exp\big((\mu - \sigma^2/2)t + W(t)\big),$$

Should say:

$$S(t) = S(0)\exp\big((\mu - \sigma^2/2)t + \sigma W(t)\big),$$

- Page 8, line 7, says:

$$S(t) = S(0)\exp\big((r - \sigma^2/2)t + W(t)\big),$$

Should say:

$$S(t) = S(0)\exp\big((r - \sigma^2/2)t + \sigma W(t)\big),$$

- Page 13, line 7, says:

$$\sigma_1 = \sigma_0 + \frac{C(\sigma_0) - QP}{S(0)\sqrt{T}\phi(d_1)}.$$

Should say:

$$\sigma_1 = \sigma_0 - \frac{C(\sigma_0) - QP}{S(0)\sqrt{T}\phi(d_1)}.$$

- Page 14, line 4, says:

$$\sigma_1 = 0.15 + \frac{495 - 460}{2029} = 0.2216$$

Should say:

$$\sigma_1 = 0.15 + \frac{495 - 640}{2029} = 0.2216$$

- Page , line , says:

$$r(t,T) = \frac{1}{T-t}\int_t^T r(s)ds, \qquad \bar{\sigma}(t,T)^2 = \frac{1}{T-t}\int_t^T \sigma(s)ds.$$

Should say:

$$\bar{r}(t,T) = \frac{1}{T-t}\int_t^T r(s)ds, \qquad \bar{\sigma}(t,T)^2 = \frac{1}{T-t}\int_t^T \sigma(s)^2 ds.$$

- Example in pages 18 and 19 is supressed.

15 Calibration in Black Scholes Model and Binomial Trees (second part)

- Page 8, line 3, says:

$$\mathbf{E}(1+X) = u^2 p + (1-p)d^2$$

Should say:

$$\mathbf{E}(1+X)^2 = u^2 p + (1-p)d^2$$

- Page 16, line 1, says: First we compute the Call Option price with the Binomial Tree formula:

  Should say: First we compute the Put Option price with the Binomial Tree formula:

16 Time dependence in Black Scholes

- Page 15, line -5, says:

  $$C(S(0); K; r; \sigma) = QP,$$

  Should say:

  $$C(S(0); K; T; r; \sigma) = QP,$$

17 The volatility Smile and its Implied Tree

- Page 5, line -4, says: A downward movement to a value S_B with probability $1 - p$,

  Should say: A downward movement to a value S_B with probability $1 - p_1$,

- Page 8, line -1, says:

  $$\sigma = 19.54 + 16.5\left[\frac{19.47 - 19.54}{200}\right] = 19.52$$

  Should say:

  $$\sigma = 19.54 + 16.5\left[\frac{19.47 - 19.54}{200}\right] = 19.534$$

- Page 9, line 2, says:

  $$Call = e^{-rT} p(S_A - S_0),$$

  Should say:

  $$Call = e^{-rT} p_1(S_A - S_0),$$

- Page 9, line 4, says:

  $$\frac{FS_A - S_0^2}{S_A + S_0} = e^{rT} Call,$$

  Should say:

  $$\frac{F_1 S_A - S_0^2}{S_A + S_0} = e^{rT} Call,$$

18 Diffusion processes for stocks and interest rates

- Page 4, line -2, says:

$$X(t+\Delta) = \alpha\,\Delta + \beta\,\Delta W,$$

  Should say:

$$X(t+\Delta) = x + \alpha\,\Delta + \beta\,\Delta W,$$

- Page 6, lines 7 to 10 , says:
  - If $X(t) < b/a$ then the drift α is positive, and the proces tends to go up,
  - If $X(t) > b/a$ then the drift is negative, and the process tends to go down.

  Should say:
  - If $X(t) < a/b$ then the drift α is positive, and the proces tends to go up,
  - If $X(t) > a/b$ then the drift is negative, and the process tends to go down.

21 Option Pricing for Diffusion with Jumps

- Page 3, line -2, says:
  - Y_i is the jump of the log-stock price, assumed to be lognormal with parameters (ν, δ),

  Should say:
  - Y_i is the jump of the log-stock price, assumed to be normal with parameters (ν, δ),

22 Fixed Income Finance

- Page 6, line -1, says: ... the cupon will pay 7.27 points.

  Should say: ... the cupon will pay 5.47 points

- Page 17, line 2, says:

$$V = 100\, e^{y(T-t)} + C \times \sum_k e^{y(t_k - t)}.$$

  Should say:

$$V = 100\, e^{-y(T-t)} + C \times \sum_k e^{-y(t_k - t)}.$$

24 Pricing Fixed Income Derivatives through Black's Formula

- Page 8, line 8,9,10, says: Example. Compute the Bond Call Option price under the following characteristics. Thea 10.month European call option on a The underlying is a 9.75 year Bond with a face value of $1,000.

  Should say: Example. Compute the Bond Call Option price under the following characteristics The underlying is a 9.75 year Bond with a face value of $1,000.

25 Interest rates models

- Page 18, line 3, says: Option prices can be computed also as

  Should say: Zero coupon bond prices can be computed also as