2 Sumas de Gauss.

$q>2$ primo

$$G = \sum_{m=1}^{q-1} \left(\frac{m}{q}\right) e_q(m)$$

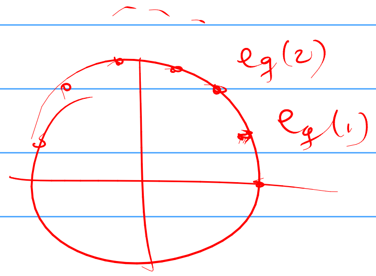
$$\boxed{e_q(x) = e\left(\frac{x}{q}\right) = e^{2\pi i x/q}}$$

Prop
$$G^2 = \begin{cases} q & q \equiv 1\ (4) \\ -q & q \equiv 3\ (4) \end{cases}$$

¿Signo de G? $\quad e_q(m) = e_q(1)^m \quad e_q(1) \sim \zeta$

$$G_\zeta = \sum_{m=1}^{q-1} \left(\frac{m}{q}\right) \zeta^m$$

Obs: signo depende de ζ!



Teo (Gauss)
$$G = \begin{cases} q^{1/2} & q \equiv 1\ (4) \\ i\, q^{1/2} & q \equiv 3\ (4) \end{cases}$$

$$\underline{\underline{q^{1/2} > 0}}$$

$$\sum_{0}^{q-1} e_q(x) = 0$$
$$\parallel$$
$$1 + \sum_R + \sum_N = 0$$

$$G = \sum e_q(R) - \sum e_q(N)$$
$$= 1 + 2\sum e_q(R)$$
$$= \sum_{x=0}^{q-1} e_q(x^2)$$

$$S = S_N = \sum_{n=0}^{N-1} e_N(n^2) \overset{\text{Dirichlet}}{=} \begin{cases} (1+i)\, N^{1/2} & N \equiv 0\ (4) \\ N^{1/2} & N \equiv 1\ (4) \\ 0 & N \equiv 2\ (4) \\ i\, N^{1/2} & N \equiv 3\ (4) \end{cases}$$

Teorema Sumación de Poisson

$f: \mathbb{R} \to \mathbb{C}$ "buena" (e.g. continua monótona a trozos)

$$\sum_{n=A}^{B}{}' f(n) = \sum_{\nu=-\infty}^{\infty} \int_A^B f(x)\, e(\nu x)\, dx$$

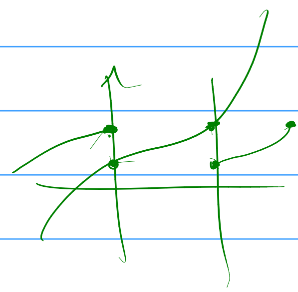
en vez de $f(A)$, sumamos $\frac{1}{2} f(A)$
$f(B)$ " $\frac{1}{2} f(B)$

$\{x\} = x - \lfloor x \rfloor$

Dem $A = 0$, $B = 1$ $f_0(x) = f(\{x\})$ periódica, disc. en 0

Fourier $f_0(x) \doteq \sum_{\nu=-\infty}^{\infty} a_\nu\, e(\nu x)$ (ojo con discontinuidades)

$$a_\nu = \int_0^1 f(x)\, e(-\nu x)\, dx$$



$x = 0$

$$\frac{1}{2}\left(f(0) + f(1)\right) = \sum a_\nu = \sum_{\nu=-\infty}^{\infty} \int_0^1 f(x)\, e(\nu x)\, dx$$

En general $f_n(x) = f(n + \{x\})$

$$\hookrightarrow \frac{1}{2}\left(f(n) + f(n+1)\right) = \sum_{\nu=-\infty}^{\infty} \int_n^{n+1} f(x)\, e(\nu x)\, dx$$

El resultado sale de sumar $n = A, \dots, B-1$ #

$$S_N = \sum_{n=0}^{N} e_N(n^2) \qquad f(x) = e_N(x^2) = e\left(\frac{x^2}{N}\right)$$

Poisson

$$= \sum_{\nu=-\infty}^{\infty} \int_0^N e\left(\nu x + \frac{x^2}{N}\right) dx$$

$x = Nu$

$$= N \sum_{\nu=-\infty}^{\infty} \int_0^1 e\Big(N(\underbrace{u^2 + \nu u}_{(u+\frac{1}{2}\nu)^2 - \frac{1}{4}\nu^2})\Big)\, du$$

$y = u + \frac{1}{2}\nu$

$$= N \sum_{\nu=-\infty}^{+\infty} \underbrace{e\left(-\frac{1}{4}N\nu^2\right)} \int_{\frac{1}{2}\nu}^{\frac{1}{2}\nu+1} e(Ny^2)\, dy$$

$$e_4(-N\nu^2) = \begin{cases} 1 & \nu = 2\mu \\ i^{-N} & \nu = 2\mu+1 \end{cases}$$

$$= N \sum_{\mu=-\infty}^{\infty} \int_{\mu}^{\mu+1} e(Ny^2)\, dy + N i^{-N} \sum_{\mu=-\infty}^{\infty} \int_{\mu+\frac{1}{2}}^{\mu+\frac{1}{2}+1} e(Ny^2)\, dy$$

$$S_N = N(1 + i^{-N}) \underbrace{\int_{-\infty}^{\infty} e(Ny^2)\, dy}_{N^{-1/2} \cdot C} \qquad u = N^{1/2} y$$

$$S_N = (1+i^{-N})\, N^{1/2} \cdot C$$

$$N=1 \rightsquigarrow S_1 = 1 \rightsquigarrow C = (1+i^{-1})^{-1}$$

$$\Rightarrow \quad S_N = \underbrace{\left(\frac{1+i^{-N}}{1+i^{-1}}\right)}_{\text{depende de } N \text{ mód } 4} \cdot N^{1/2}$$