Fórmula de Dirichlet (2)

$d < 0$ fundamental. f FCB, disc $f = d$. (definida +)

$$(\otimes)\begin{cases} R_f(n) = \#\{(x,y)\in\mathbb{Z}^2 : f(x,y) = n\} < \infty \\ R(n) = R_d(n) = \sum_{f/\sim} R_f(n). \end{cases}$$

$\boxed{\text{Teo}}$ $n>0$, $(n,d)=1 \Rightarrow R_d(n) = w_d \cdot \sum_{m|n} \left(\frac{d}{m}\right)$

$$\Rightarrow \lim_{N\to\infty} \frac{1}{N} \sum_{\substack{n=1\\(n,d)=}}^{N} R_d(n) = w_d \frac{\varphi(|d|)}{|d|} \cdot L(1,\chi_d)$$

Hoy $\displaystyle\lim \frac{1}{N}\sum_{\substack{n=1\\(n,d)}}^{N} R_f(n) = ?$

$$\sum_{\substack{n=1\\(n,d)=1}}^{N} R_f(n) = \#\{(x,y) : 1\le f(x,y)\le N, \underbrace{(f(x,y),d)=1}_{\circledast}\}$$

La condición $\circledast$ solo depende de (x,y) mod d.

$$\sum_{\substack{x_0,y_0 \,(\text{mód } d)\\ (f(x_0,y_0),d)=1}} \underbrace{\#\{(x,y)\equiv(x_0,y_0) \text{ mód } d \;/\; f(x,y)\le N\}}$$

Ejercicio

$$\#\{(x_0,y_0) \bmod d \ : \ (f(x_0,y_0), d) = 1\} = |d| \cdot \varphi(|d|)$$

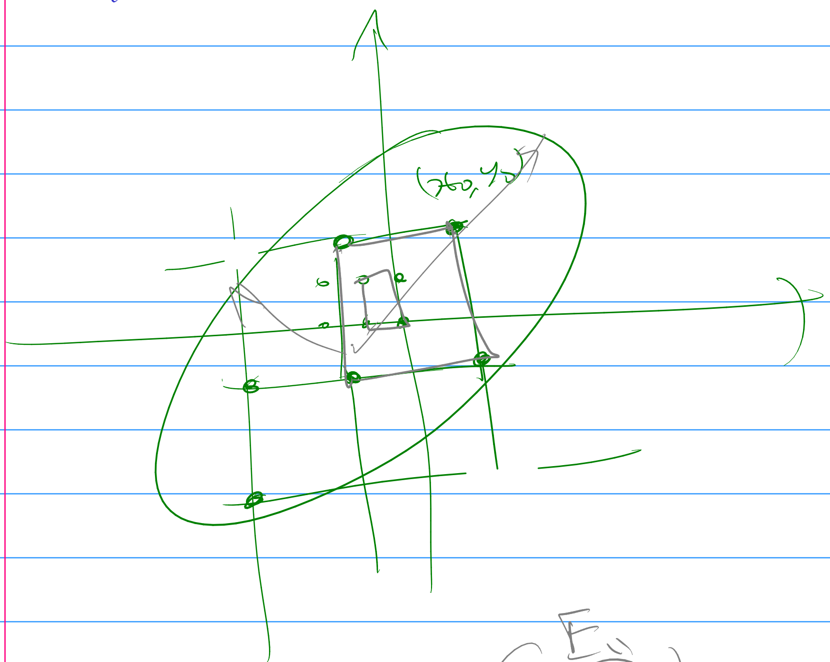
(Ideas: ① basta probarlo para $d = p^e$

sup. $p \nmid a$

→ p impar $\quad 4af = (2ax+by)^2 - dy^2$

→ $p=2$ ..... $x^2+y^2 \le N$)

$$\#\{(x,y) \equiv (x_0,y_0) : \underbrace{f(x,y) \le N}_{\text{elipse}}\}$$



$N \to \infty$

Ej ① $\quad$ Área$(E_N) = \frac{2\pi}{\sqrt{|d|}} \cdot N$

(a) rotar ⇝ $ax^2 + cy^2 \le N \qquad \frac{\pi N}{\sqrt{ac}}$

② $\# E \cap (\mathbb{Z}\times\mathbb{Z}) \sim$ Área $+ \ O(\sqrt{N})$

(3) $\# E \cap \{(x,y) \equiv (x_0,y_0)\} \sim \frac{\text{Area}}{(d)^2} + O(\sqrt{N})$

$$\frac{1}{N} \sum_{\substack{n=1 \\ (n,d)=1}}^{N} R_f(n) = \frac{1}{N} |d| \cdot \varphi(|d|) \cdot \left(\frac{2\pi}{\sqrt{|d|}\,|d|^2} N + O(\sqrt{N}) \right)$$

$$= \frac{\varphi(|d|)}{|d|} \frac{2\pi}{\sqrt{|d|}} + O(N^{-1/2})$$

$$h(d) \cdot \cancel{\frac{\varphi(|d|)}{|d|}} \cdot \frac{2\pi}{\sqrt{|d|}} = \omega_d \cancel{\frac{\varphi(|d|)}{|d|}} \cdot L(1,\chi_d)$$

Teo

$$\boxed{h(d) = \frac{\omega_d}{2} \cdot \frac{\sqrt{|d|} \cdot L(1,\chi_d)}{\pi}} \quad (d<0)$$

$$\frac{\zeta(2)}{\pi^2} = \frac{1}{6} \qquad \frac{\zeta(2k)}{\pi^{2k}} \in \mathbb{Q}$$

Caso $d>0$ $\longrightarrow$ hay ∞ autometrías.

$\Updownarrow$

soluciones de $t^2 - du^2 = 4$ Ec. de Pell

$$\boxed{\varepsilon} = \frac{t_0 + u_0\sqrt{d}}{2} \rightsquigarrow N(\varepsilon) = 1$$

$$\pm\varepsilon^m = \frac{t + u\sqrt{d}}{2} \rightsquigarrow N(\pm\varepsilon^m) = 1$$

$$\longrightarrow R_f(n) = \#\{(x,y) : f(x,y) = n\} / \mathrm{Aut}(f) < \infty$$

$f(x,y) = n$

$\downarrow$

$f(\gamma(x,y)) = n$

$$\longrightarrow \boxed{h(d) = \frac{d^{1/2} \cdot L(1,\chi_d)}{\log \varepsilon}}$$

FCB disc d $\longleftrightarrow$ $\left\{\begin{array}{l}\text{Reales en} \\ [K:\mathbb{Q}]=2 \\ \operatorname{disc} K = d\end{array}\right.$

Autometrías $\longleftrightarrow$ unidades

Teo

- $\circledast$ $L(1,\chi_d) = \dfrac{-\pi}{|d|^{1/2}} \displaystyle\sum_{m=1}^{|d|} m\left(\frac{d}{m}\right)$, $d<0$
- $\circledast$ $L(1,\chi_d) = -\dfrac{1}{d^{1/2}} \displaystyle\sum_{m=1}^{d} \left(\frac{d}{m}\right) \log \operatorname{sen}\left(\frac{m\pi}{d}\right)$, $d>0$

$\rightsquigarrow$ $\boxed{h(d) = \Sigma \text{ finita}}$

$K/\mathbb{Q}$ extensión finita $\rightsquigarrow$ $\zeta_K(s)$ (función zeta de Dedekind)

Fórmula del número de clases:

$$\operatorname{res}_{s=1} \zeta_K(s) \longleftrightarrow h(K)\cdot \operatorname{reg}(K)$$

$K/\mathbb{Q}$ cuadrático disc d:

$$\zeta_K(s) = \zeta(s)\cdot L(s,\chi_d)$$

$$\mathrm{Res} = \underset{\overset{\shortparallel}{1}}{\mathrm{res}} \times L(1,\chi_d)$$

K imag $\quad \operatorname{reg} = 1 \quad (d<0)$

K real $\quad \operatorname{reg} = \log \varepsilon \quad (d>0)$

$$\boxed{\mathbb{Z}[i]}$$

$$p = x^2+y^2 = (x+iy)\cdot(x-iy)$$

$$x^n+y^n=z^n \rightarrow \boxed{x^n = (y-z)(y-\xi z)\cdots(\ )}$$

$$\boxed{\mathbb{Q}(\xi)}$$