§2 Funciones aritméticas

Apostol cap 2, 3

$$f : \mathbb{Z}_{>0} \longrightarrow \mathbb{C}$$

① Función de Möbius

Def $\mu(1) = 1$

$$\mu(n) = \begin{cases} (-1)^t & \text{si } n = p_1 \cdots p_t \text{ primos distintos} \\ 0 & \text{si no.} \end{cases}$$

n:	1	2	3	4	5	6	7	8	9	10
$\mu(n)$:	+1	−1	−1	0	−1	+1	−1	0	0	+1

Prop $$\sum_{d \mid n} \mu(d) = \delta(n) = \begin{cases} 1 & n = 1 \\ 0 & n > 1 \end{cases}$$

(Ap: $\delta \to I$)

Dem $n = p_1^{a_1} \cdots p_t^{a_t} > 1$

$$\sum_{d \mid n} \mu(d) = \mu(1) + \sum_i \mu(p_i) + \sum_{i<j} \mu(p_i p_j) + \cdots$$

$$= 1 - \binom{t}{1} + \binom{t}{2} - \binom{t}{3} + \cdots + (-1)^t \binom{t}{t}$$

$$= (1-1)^t = 0$$

(2) **Función de Euler:**

$$\varphi(n) = \sum_{\substack{i=1 \\ (i,n)=1}}^{n} 1 = \sum_{i=1}^{n} \delta((i,n))$$

Prop $\sum_{d|n} \varphi(d) = n$

Rem $S = \{1, \dots, n\} = \dot{\bigcup}_{d|n} A(d)$ (*)

$$A(d) = \{i = 1, \dots, n : (i,n) = d\}$$

Ej. $\#A(d) = \varphi(n/d)$

$$\sum_{d|n} \varphi(d) = \sum_{d|n} \varphi(n/d) \overset{*}{=} \#S = n. \quad \#$$

Prop $\varphi(n) = \sum_{d|n} \mu(n) \cdot \frac{n}{d}$

Rem

$$\varphi(n) = \sum_{i=1}^{n} \delta((i,n)) = \sum_{i=1}^{n} \sum_{d|(i,n)} \mu(d)$$

$$= \sum_{d|n} \sum_{q=1}^{n/d} \mu(d) = \sum_{d|n} \left(\mu(d) \underbrace{\sum_{q=1}^{n/d} 1}_{i = q \cdot d} \right) = \sum_{d|n} \mu(d) \frac{n}{d}.$$

Def Producto de Dirichlet

$$(f * g)(n) = \sum_{d \mid n} f(d) \cdot g\left(\frac{n}{d}\right)$$

Las props:

$1(n) = 1 \ \forall n$

$N(n) = n$

① $\mu * 1 = \delta$

② $\varphi * 1 = N$

③ $\mu * N = \varphi$.

Ejercicio ① $*$ es comut. asoc.

δ neutro

② ¿cuándo hay inverso?