Apostol capítulo 3

Promedios de funciones aritméticas

$$f(n) \rightsquigarrow \bar{f}(n) := \frac{1}{n}\sum_{k=1}^{n} f(k).$$

ejemplo $\bar{d}(n) \sim \log n$.

Teo (Dirichlet) $\displaystyle\sum_{k\le x} d(k) = x\log x + (2\gamma-1)x + O(\sqrt{x})$

$O(x^\alpha)$

$$\gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}-\log n\right) \approx 0.577\ldots$$

Teo fórmula de sumación de Euler. $\{t\} = t - \lfloor t \rfloor$

$\boxed{0<y<x}$

f' continua en $[y,x]$

$$\sum_{y<n\le x} f(n) = \int_y^x f(t)\,dt + \int_y^x \{t\} f'(t)\,dt - \{x\}f(x) + \{y\}f(y)$$

Corolario $\displaystyle\sum_{n\le x}\frac{1}{n} = \log x + \gamma + O\left(\frac{1}{x}\right)$

Dem $y=1$, $f(t)=\frac{1}{t}$

$$\int_1^\infty - \boxed{\int_x^\infty}_{=O(1/x)}$$

$$\sum_{n\le x}\frac{1}{n} = 1 + \overbrace{\int_1^x \frac{1}{t}\,dt}^{\log x} - \int_1^x \frac{\{t\}}{t^2}\,dt - \underbrace{\frac{\{x\}}{x}}_{=O(1/x)}$$

$$= \log x + \overbrace{\left(1-\int_1^\infty \frac{\{t\}}{t^2}\,dt\right)}^{\gamma} + O(1/x)$$