Apostol capítulo 4

Distribución de los números primos

Objetivo: $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$ (TNP)

→ Selberg y Erdős (1949)

→ Hadamard
→ De la Vallée Poussin } (1896) Variable Compleja

Chebyshev $\psi(x) = \sum_{n\le x} \Lambda(x)$.

(PNT $\iff \frac{\psi(x)}{x} \to 1$)

$$* \quad \psi(x) = \sum_{m=1}^{\infty\ \log_2 x} \underbrace{\sum_{p\le x^{1/m}} \log p}$$

(Si $x < 2^m$ la suma es vacía)

$$\theta(x) = \sum_{p\le x} \log p$$

Nota: $\psi(x) = \sum_{m=1}^{\log_2 x} \theta(x^{1/m})$

Vamos a ver: $\lim \left(\frac{\psi(x)}{x} - \frac{\theta(x)}{x} \right) = 0$

Teorema (Abel) $\quad A(x) = \sum_{n \le x} a(n)$

Sea $f: [y, x] \to \mathbb{C}$ con derivada continua.

$(0 < y < x)$

$$\sum_{y < n \le x} a(n) f(x) = A(x) f(x) - A(y) f(y) - \int_y^x A(t) f'(t) dt$$

$$\| \quad \int_y^x f(t)\, dA(t) \qquad \text{Riemann-Stieltjes}$$

Prop $(x \ge 2)$

(1) $$\theta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$$

(2) $$\pi(x) = \frac{\theta(x)}{\log x} + \int_2^x \frac{\theta(t)}{\log^2 t} dt$$

Cor: $\theta(x) \sim x \iff \pi(x) \sim \frac{x}{\log x}$

(1) $a(n) = \begin{cases} 1 & n \text{ es primo} \\ 0 & \text{---} \end{cases}$ $\quad (A(x) = \pi(x))$ $\quad \to \theta(x) = \sum a(n) \log n$

$[b(n) = a(n) \log n]$