15 $N(T)$.

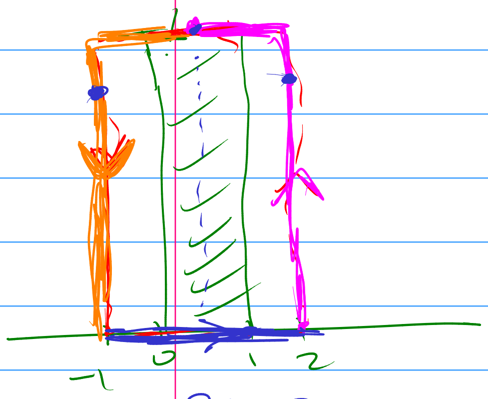
$$N(T) = \#\{s : 0<\sigma<1,\ 0<t<T,\ \xi(s)=0\}$$

$\overset{\shortparallel}{\sigma+it}$

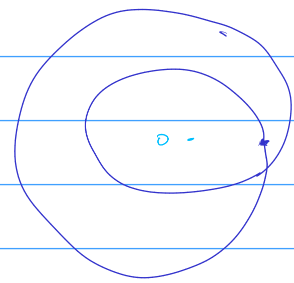
Riemann conj. asintótica para $N(T)$, von Mangoldt demuestra.

$\xi(s)$ sup $T \gg 0$, no tiene cero con $\operatorname{Im} s = T$

Ppio del argumento: $N(T) = \dfrac{1}{2\pi} \underbrace{\Delta_R \arg \xi(s)}_{= \int_R d \arg \xi(s)}$



$$= \frac{1}{2\pi i} \oint_R \frac{\xi'(s)}{\xi(s)}\, ds$$



R: $2 \to 2+iT \to -1+iT \to -1 \to 2$.

$-1 \to 2$ $\quad \xi(s)$ real no nula $\to \Delta \arg = 0$

L: $2 \to 2+iT \to \frac{1}{2}+iT$

L': $\frac{1}{2}+iT \to -1+iT \to -1$

$$\xi(\sigma+it) = \xi(1-\sigma-it) = \overline{\xi(1-\sigma+it)}$$

$\Delta_L = \Delta_{L'}$.

$$\boxed{N(T) = \frac{1}{\pi}\Delta_L \arg \xi(s)}$$

$$\xi(s) = (s-1)\cdot \pi^{-s/2}\cdot \Gamma(s/2+1)\cdot \zeta(s)$$

$$\Delta_L \arg(s-1) = \arg(iT - 1/2) = \frac{\pi}{2} + O(1/T)$$

$$\Delta_L \arg \pi^{-s/2} = \Delta_L\left(-\frac{t}{2}\log\pi\right) = -\frac{T}{2}\log\pi .$$

$$\Delta_L \arg\Gamma\left(\frac{s}{2}+1\right) = \operatorname{Im}\log\Gamma\left(\frac{iT}{2}+\frac{5}{4}\right)$$

$$= \operatorname{Im}\left(\left(\frac{iT}{2}+\frac{3}{4}\right)\overbrace{\underbrace{\log\left(\frac{iT}{2}+\frac{5}{4}\right)}_{Re}}^{Im} - \frac{iT}{2}\overbrace{-\frac{5}{4}-\frac{1}{2}\log 2\pi}^{real.}\right) + O\left(\frac{1}{T}\right)$$

Stirling: $\log\Gamma(s) = (s-1/2)\log s - s + \frac{1}{2}\log 2\pi + O(1/s)$

$$= \frac{T}{2}\log\frac{T}{2} - \frac{T}{2} + \frac{3}{8}\pi + O(1/T)$$

$$\to N(T) = \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi} + \overbrace{\frac{7}{8} + \frac{S(T)}{\pi} + O(1/T)}^{O(\log T)}$$

donde: $S(T) = \Delta_L \arg\xi(s) = $ " $\arg\zeta(1/2+iT)$ "

Afir: $S(T) = O(\log T)$

Littlewood: $\int_0^X S(T)\,dT = O(\log X)$ ← MUCHA CANCELACIÓN

Lema $\rho = \beta + i\gamma$ ceros de ξ no triviales

$T >> 0$

$$\Rightarrow \sum_{\rho} \frac{1}{1+(T-\gamma)^2} = O(\log T).$$

ejercicio

Consecuencias:

(a) # ceros $T-1 < \gamma < T+1$ es $O(\log T)$

(b) $\sum' (T-\gamma)^{-2}$ (fuera de) es $O(\log T)$

(c) $$\frac{\xi'(s)}{\xi(s)} = \sum_{\substack{\rho \\ |t-\gamma|<1}} \frac{1}{s-\rho} + O(\log t)$$

$s = \sigma + it$
$-1 \le \sigma \le 2$
$t >> 0$
$t \ne \gamma$

Aplicar fórmula para $\frac{\xi'(s)}{\xi(s)}$

en $\begin{cases} \sigma + it \\ 2+it \end{cases}$ restar

ejercicio

$$\dashrightarrow \frac{\xi'(s)}{\xi(s)} = O(\log t) + \sum_{\rho} \left(\frac{1}{s-\rho} - \frac{1}{2+it-\rho} \right)$$

$|t-\gamma| \ge 1$ ------- $\sum \le \dots \le \sum \frac{3}{|\gamma-t|^2} = O(\log t)$ (b)

$|t-\gamma| < 1$ --- $\left|\frac{1}{2+it-\rho}\right| \le 1$ y son $O(\log t)$ términos (a).

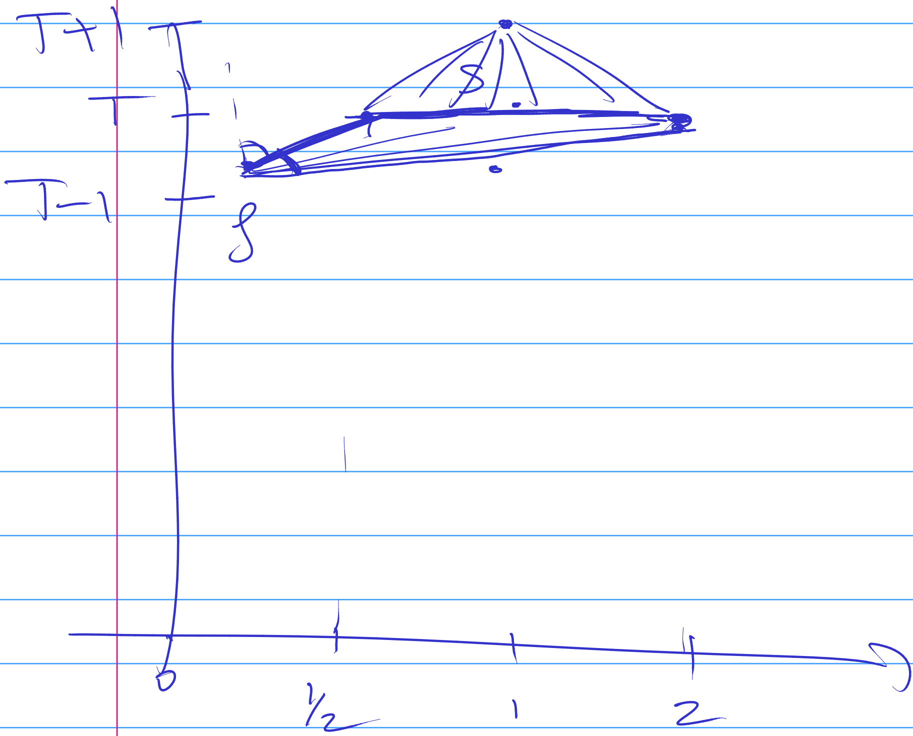
(d) $S(T) = O(\log T)$

Dem

$$S(T) = \underbrace{\int_2^{2+iT} \operatorname{Im}\left(\frac{\zeta'(s)}{\zeta(s)}\right) ds}_{= O(1)} - \underline{\int_{1/2+iT}^{2+iT} \operatorname{Im}\left(\frac{\zeta'(s)}{\zeta(s)}\right) ds}$$

$$\left| \int_{1/2+iT}^{2+iT} \operatorname{Im}(s-\rho)^{-1} ds \right| = \left| \Delta \arg(s-\rho) \right| \leq \pi$$

$\operatorname{Im}(\rho) \in (T-1, T+1)$



$\downarrow$ por (a)

hay $O(\log T)$ términos

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