Teo $N(T) = \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi} + O(\log T)$.

Cor $\rho_j = \beta_j + i\gamma_j \qquad \gamma_1 \le \gamma_2 \le \gamma_3 \le \dots$

$\Rightarrow \gamma_n \sim \frac{2\pi n}{\log n}$

Dem $2\pi N(T) \sim T\log T \to 2\pi N(\gamma_n \pm 1) \sim (\gamma_n \pm 1)\log(\gamma_n \pm 1)$

$N(\gamma_n - 1) \le n \le N(\gamma_n + 1) \qquad \sim \gamma_n \log\gamma_n$

$\Rightarrow 2\pi n \sim \gamma_n \log\gamma_n \Rightarrow \log n \sim \log\gamma_n$

$\gamma_n \log n$

$\Rightarrow \gamma_n \sim \frac{2\pi n}{\log n}$. #

Littlewood ① $\int_0^X \left(N(T) - \frac{T}{2\pi}\log\frac{T}{2\pi} + \frac{T}{2\pi} - \frac{7}{8}\right) dX$

$\left(\int_0^X S(T) = O(\log X)\right) \qquad = O(\log X)$.

② $\gamma_{n+1} - \gamma_n \to 0$

(Dem. en Titchmarsh - "The theory of the Riemann zeta-function")

(16) χ primitivo mód q.

$N(T,\chi)$ = # ceros de $L(s,\chi)$ en

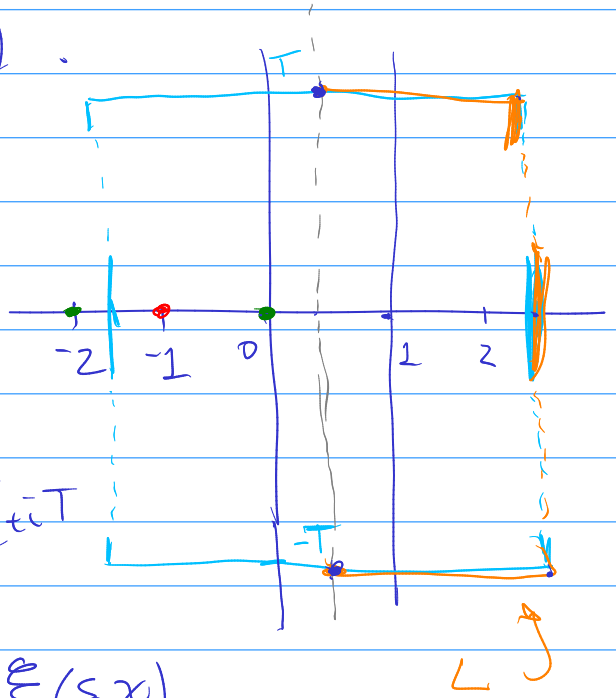
$0<\delta<1$ $\quad |t|<T$ $\quad \mathcal{L}$

Teo $\frac{1}{2}N(T,\chi) = \frac{T}{2\pi}\log\frac{qT}{2\pi} - \frac{T}{2\pi} + O(\log T + \log q)$

Dem similar a la de $N(T)$.

$\chi(-1)=1$ $(a=0)$ $\Gamma(s/2)$

$\chi(-1)=-1$ $(a=1)$ $\Gamma\left(\frac{s+1}{2}\right)$



R: $\frac{5}{2}-iT \to \frac{5}{2}+iT \to \frac{-3}{2}+iT \to \frac{-3}{2}-iT \to \frac{5}{2}+iT$

$\hookrightarrow 2\pi\left(N(T,\chi)+1\right) = \Delta_{\mathcal{L}} \arg \xi(s,\chi)$

Ec. func: $\xi(s,\chi) = w\,\xi(1-s,\bar{\chi})$ $\quad |w|=1$

$= w\,\xi(1-\bar{s},\chi)$

$\to \arg \xi(\sigma+it,\chi) = \arg w - \arg \xi(1-\sigma+it,\chi)$ (de)

$\Delta_R = 2\Delta_{\mathcal{L}}$

$\Delta_R \arg\left(\frac{q}{\pi}\right)^{\frac{s+a}{2}} = T\cdot\log\left(\frac{q}{\pi}\right)$

$\Delta_R \arg \Gamma\left(\frac{s+a}{2}\right) = T\log\frac{T}{2} - T + O(1)$

Prop $\Delta_L \arg L(s,\chi) = O(\log T + \log q)$

Lema $\sum_\rho \frac{1}{1+(t-\gamma)^2} = O(\mathcal{L}_\chi)$ $\quad \mathcal{L}_\chi = \log q + \log(|t|+2)$

ejercicio

$\rightsquigarrow$ (a) # sumandos con $|t-\gamma|<1$ es $O(\mathcal{L}_\chi)$

(b) sum $|t-\gamma|\geq 1$ es $O(\mathcal{L}_\chi)$

(c) $\frac{L'(s,\chi)}{L(s,\chi)} = \sum_{|t-\gamma|<1} \frac{1}{s-\rho} + O(\mathcal{L}_\chi)$

sumandos $O(\mathcal{L}_\chi)$

(d) $\int_L \frac{L'(s,\chi)}{L(s,\chi)} ds = O(\log q + \log T)$

$$= \int_{5/2-iT}^{5/2+iT} = O(1).$$

$$\int_{1/2\pm iT}^{5/2\pm iT} \frac{1}{s-\rho} ds = \cdots = O(1) \quad \#$$

χ no primitivo ok. Pero ceros con $\mathrm{Re}\, s = 0$

$(1-\chi_1(p)p^{-s})$

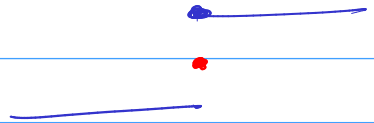
$$N_R(T,\chi) = \frac{T}{\pi}\log\frac{T}{2\pi} + O(T\log q)$$

Adelanto de las próximas clases...

17, 18 $\pi(x) = \dots$

$$\psi(x) = \sum_{n \le x} \Lambda(n) = \sum_{p^n \le x} \log p .$$

$$\psi_0(x) = \psi(x) - \frac{\Lambda(x)}{2}$$



Teo (von Mangoldt) "pasto"

$$\psi_0(x) = x - \sum_{\rho} \frac{x^\rho}{\rho} - \frac{\zeta'(0)}{\zeta(0)} - \frac{1}{2}\log(1-x^{-2})$$

$\log 2\pi$

"suma simétrica"

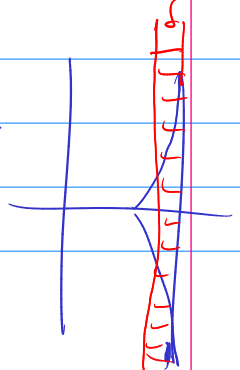
$$\left(\lim_{T \to \infty} \sum_{|\gamma| < T}{}'' \frac{x^\rho}{\rho} \right)$$

$$\sum_{\omega}{}'' \frac{x^\omega}{\omega}$$

$\omega = -2, -4, -6, \dots$

ceros triviales

Teo + N(T) + no ceros en Res = 1 $\longrightarrow$ $\psi_0(x) \sim x$.



+ no ceros en "región" $\longrightarrow$ $|\psi_0(x) - x| \le$?

$O(x^{1-\varepsilon})$

+ H.R. $\longrightarrow$ $|\psi_0(x) - x| = O(x^{1/2+\varepsilon})$